Exam 2 Practice #1

Problem 1: Find $\frac{\partial z}{\partial x}$ if z is defined implicitly as a function of x and y by the equation

$$x^4 + y^4 + z^4 + 8xyz = 100$$

(a) $-\dfrac{x^3 + 2yz}{z^3 + 2xy}$ (b) $-\dfrac{x^4 + 2yz}{y^4 + 2xy}$ (c) $-\dfrac{x^2 + 2yz}{z^2 + 2xy}$ (d) $\dfrac{x^3 + 2yz}{z^3 + 2xy}$ (e) $-\dfrac{x^3 + 8xz}{z^3 + 8xy}$

Problem 2: If $z = e^x \sin y$, let z be a function of s and t via substitution where $x = st^2$ and $y = s^2 t$. Find $\frac{\partial z}{\partial s}$ at $s = 0$ and $t = 1$.

(a) 0 (b) 2 (c) 1 (d) -1 (e) -2

Problem 3: If $f(x, y) = xe^y$, find $\nabla f(1, 0)$.

(a) $\langle 1, 1\rangle$ (b) $\langle 1, 0\rangle$ (c) $\langle 0, 1\rangle$ (d) $\langle 2, 1\rangle$ (e) $\langle 1, 2\rangle$

Problem 4: Find the maximum rate of change of $f(x, y, z) = \frac{x^2}{2} + 2\sin y + z^2$ at $(1, 0, -1)$.

(a) 3 (b) 2 (c) $\sqrt{2}$ (d) 1 (e) -1

Problem 5: Find the directional derivative of the function $f(x, y, z) = \sqrt{xyz}$ at $(6, 3, 2)$ in the direction $\vec{v} = \langle 2, -1, -2\rangle$. Note $\vec{v}$ has length 3.

(a) -1 (b) 1 (c) 2 (d) 3 (e) 6

Problem 6: Find the unit vector in the direction of the maximum rate of change of $f(x, y, z) = x^2 - y^2 - z^2$ at $(3, -2, -6)$.

(a) $\frac{1}{7}\langle 3, 2, 6\rangle$ (b) $\frac{1}{7}\langle 3, -2, -6\rangle$ (c) $\frac{1}{7}\langle 3, -2, 6\rangle$ (d) $\frac{1}{7}\langle -3, -2, 6\rangle$ (e) $\frac{1}{7}\langle -3, 2, -6\rangle$

Problem 7: Let $f(x, y, z) = x^4 + 2xy + y^4 + xz + z^3$ and $g(x, y, z) = 2x + 3y + 5z$. The surfaces $f(x, y, z) = 9$ and $g(x, y, z) = 18$ intersect in a curve. The point $(0, 1, 2)$ lies on the intersection curve. Which vector below is tangent to the intersection curve at this point?

(a) $\langle -16, 4, 4\rangle$ (b) $\langle 0, 1, 2\rangle$ (c) $\langle 3, -2, 0\rangle$ (d) $\langle 0, 1, 3\rangle$ (e) $\langle 4, 4, -12\rangle$

Problem 8: Find the volume of the solid S that is bounded above by $z = 16 - 3x^2 - y$ and lies above the rectangle $[0, 1] \times [0, 2] = \{0 \leqslant x \leqslant 1;\ 0 \leqslant y \leqslant 2\}$ in the xy-plane. You may assume $z \geqslant 0$ over this rectangle.

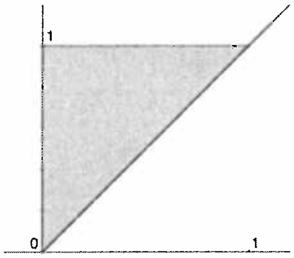
(a) 28 (b) 16 (c) 32 (d) 18 (e) -2

Problem 9: Which iterated integral below gives $\displaystyle\iint_D x\,y\;dA$ where D is the region bounded by the line $x = y - 1$ and the parabola $x^2 = 2y + 6$.

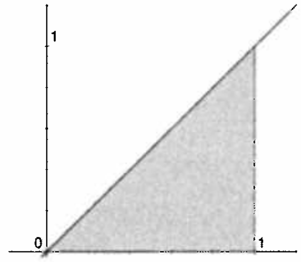
(a) $\displaystyle\int_{-2}^{4}\int_{\frac{x^2-6}{2}}^{x+1} x\,y\;dy\,dx$ (b) $\displaystyle\int_{-1}^{5}\int_{-\sqrt{2y+4}}^{y-1} x\,y\;dx\,dy$ (c) $\displaystyle\int_{-1}^{5}\int_{y-1}^{\sqrt{2y+4}} x\,y\;dx\,dy$

(d) $\displaystyle\int_{-2}^{4}\int_{x+1}^{\frac{x^2-6}{2}} x\,y\;dy\,dx$ (e) $\displaystyle\int_{-1}^{5}\int_{-2}^{4} x\,y\;dx\,dy$

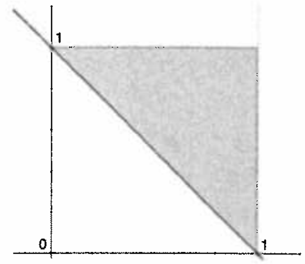
Problem 10: For which shaded region D below is $\displaystyle\iint_D e^{\frac{x}{y}}\;dA$ evaluated by the iterated integral $\displaystyle\int_0^1\int_x^1 e^{\frac{x}{y}}\;dy\,dx$?



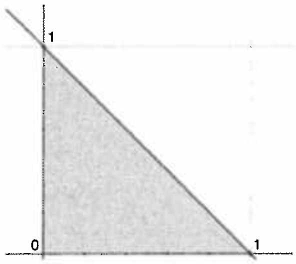
(a)



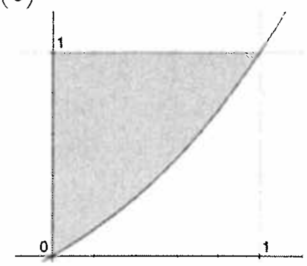
(b)



(c)



(d)



(e)

Problem 11: Let $f(x,y) = x^2y + xy^2 + 3xy$. Find the critical points of f and tell what type each one is.

Problem 12: Suppose that $(1,1)$ is the only critical point of the function $f(x,y) = 2x - 2xy + y^2 + 1$. Find the absolute maximum value of the function $f(x,y)$ on the rectangle $R = [0,2] \times [0,3] = \{(x,y) \mid 0 \leqslant x \leqslant 2,\ 0 \leqslant y \leqslant 3\ \}$.

Problem 13: Find the maximum and minimum values of $f(x,y,z) = 2x - z$ subject to $x^2 + 10y^2 + z^2 = 5$ assuming they exist.

Exam 2 Practice #2

1. Find f_{yy} for $f(x,y) = \int_y^x e^{-t^2}\,dt$.

   (a) $2ye^{-y^2}$ (b) 0 (c) $-2ye^{-y^2}$ (d) ye^{-y^2} (e) $-ye^{-y^2}$

2. Let $H = xe^{y-z^2}$, $x = 2uv$, $y = u - v$ and $z = u + v$. Find $\dfrac{\partial H}{\partial u}$ when $u = 3$ and $v = -1$.

   (a) 16 (b) 36 (c) 3 (d) -1 (e) 2

3. Let $f(x,y,z) = x\sin(yz)$. Find the directional derivative at $(1,3,0)$ in the direction $\vec{v} = \langle 1, 2, -2\rangle$.

   (a) -2 (b) 2 (c) 1 (d) 3 (e) -3

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4. Find the maximum rate of change of $f(x,y) = \sin(xy)$ at $(1,0)$.

   (a) 1 (b) -1 (c) 0 (d) $\cos 1$ (e) $\sin 1$

A5. Suppose that $(1, -1)$ is a critical point of a smooth function $f(x, y)$ with continuous second derivatives, $f_{xx}(-1, 1) = 3$, $f_{xy}(-1, 1) = 2$ and $f_{yy}(-1, 1) = 2$. What can you say about the type of the critical point of $(-1, 1)$ of f?

(a) a local minimum (b) a saddle point (c) a local maximum

(d) no information (e) absolute maximum

A6. Find an equation of the tangent plane at the point $(3, -1, 2)$ to the ellipsoid

$$\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 3.$$

(a) $\frac{2}{3}x - 2y + z - 6 = 0$

(b) $\frac{2}{3}x + 2y + z - 6 = 0$

(c) $\frac{2}{3}x - 2y + z + 6 = 0$

(d) $\frac{3}{x} - y + 2z - 6 = 0$

(e) $\frac{3(x-3)}{2} = \frac{y+1}{-2} = z - 2$

A7. Find the volume of the solid that lies under hyperbolic paraboloid $z = 4 + x^2 - y^2$ and above the square $R = [-1, 1] \times [0, 2] = \{(x, y) \mid -1 \le x \le 1, 0 \le y \le 2\}$.

(a) 12 (b) 4 (c) 3 (d) 2 (e) 1

A8. If $D = \{(x, y) \mid x \ge 0, y \ge 0, x^2 + y^2 \le 1\}$, find the integral $\int\int_D \sqrt{1 - y^2}\,dxdy$. (Hint: not to use polar coordinates).

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{2}$ (e) $\frac{\pi}{3}$

Partial Credit Problems

9. (i) Find the points on the sphere $x^2 + y^2 + z^2 = 9$ where the tangent plane is parallel to the plane $2x - y + 2z = 99$.

(ii) Find equations of normal lines to the sphere $x^2 + y^2 + z^2 = 9$ at points derived in part (i) above.

10. Find the absolute maximum and absolute minimum values of $f(x, y) = 2x - y$ on the domain $D = \{(x, y) \mid x^2 + \frac{y^2}{4} \leq 2\}$.

11. Evaluate the iterated integral $\int_0^3 \int_{-\sqrt{9-y^2}}^0 x^2 y \, dx\, dy$ by converting to polar coordinates.

Exam 2 Practice #3

1. Use a double integral to find the area enclosed by one loop of the rose $r = 2\cos 3\theta$.

(a) $\frac{\pi}{3}$ (b) 4π (c) $\frac{\pi}{2}$ (d) 3π (e) 6π

2. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 4 - x^2 - y^2$.

(a) 4π (b) $\frac{\pi}{2}$ (c) 8π (d) $\frac{\pi}{4}$ (e) 2π

3. Find the maximum value of $f(x, y, z) = xyz$ subject to $x^2 + 2y^2 + 3z^2 = 6$.

(a) $\frac{2}{\sqrt{3}}$ (b) 1 (c) $-\frac{2}{\sqrt{3}}$ (d) 0 (e) 6

4. Find the maximum volume of a rectangular box such that the sum of lengths of its 12 edges is 24.

(a) 8 (b) 12 (c) $(12)^3$ (d) 1 (e) 0

5. Find the volume of the solid bounded by the surface $z = 6 - xy$ and the plane $x = 2$, $x = -2$, $y = 0$, $y = 3$ and $z = 0$.

(a) 72 (b) 36 (c) 6 (d) 3 (e) 0

6. Find $\displaystyle\int\int_D \frac{2y}{x^2+1}\,dA$ where $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$.

(a) $\frac{1}{2}\ln 2$ (b) $\ln 2$ (c) 1 (d) 0 (e) $-\frac{1}{2}\ln 2$

7 ~~8~~. Find $\int\int_D 2xy\,dA$, where D is the triangular region with vertices $(0,0)$, $(1,2)$ and $(0,3)$.

(a) $\frac{7}{4}$ (b) $\frac{4}{7}$ (c) 2 (d) 3 (e) 0

8 ~~9~~. Evaluate the integral by reversing the order of integration $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\,dx\,dy$

(a) $\frac{52}{9}$ (b) 4 (c) $\frac{26}{9}$ (d) $\frac{26}{3}$ (e) 2

9 ~~10~~. Find $\frac{\partial z}{\partial x}$ if $x^4 + y^4 + z^4 + 4xyz = 100$.

(a) $-\frac{x^3+yz}{z^3+xy}$ (b) $\frac{x^3+yz}{z^3+xy}$ (c) $-\frac{z^3+xy}{x^3+yz}$ (d) $-\frac{z^3+xy}{x^3+yz}$ (e) $100y$

10. Let E be the largest rectangle box with edges parallel to axes that can be inscribed in the ellipsoid $9x^2 + 36y^2 + 4z^2 = 108$. Find the volume of E. (Hint: the box intersects all octants.)

11. Find the maximum and minimum values of $f(x, y, z) = yz + xy$ subject to constraints $xy = 1$ and $y^2 + z^2 = 1$.

12. Evaluate the integral $\int_0^8 \int_{y^{\frac{1}{3}}}^2 e^{x^4}\, dx dy$.