**Problem 1:** The length of the vector $\mathbf{a} = \langle 2, 6, -3 \rangle$ is
(•) 7 (∗) 8 (∗) 144 (∗) 25 (∗) −12

**Problem 2:** Suppose that $\|\mathbf{a}\| = 3$ and $\|\mathbf{b}\| = \sqrt{18}$ and angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$. Find $\mathbf{a} \bullet \mathbf{b}$.
(•) 9 (∗) 8 (∗) 18 (∗) 6 (∗) 8

**Problem 3:** Find the scalar projection of $\mathbf{a} = \langle 4, 1, 2 \rangle$ and $\mathbf{b} = \langle -2, 3, 6 \rangle$.
(•) 1 (∗) 7 (∗) 6 (∗) 4 (∗) 3

**Problem 4:** Let $\mathbf{a} = \langle 1, 2, 2 \rangle$ and $\mathbf{b} = \langle -1, 1, 4 \rangle$. Find the vector $\text{orth}_{\mathbf{a}}\mathbf{b} = \mathbf{b} - \text{Proj}_{\mathbf{a}}\mathbf{b}$.
(•) $\langle -2, 1, 2 \rangle$ (∗) $\langle 2, 1, -2 \rangle$ (∗) $\langle 2, -2, 1 \rangle$ (∗) $\langle -2, 2, 1 \rangle$ (∗) $\langle -4, 2, 2 \rangle$

**Problem 5:** If $\mathbf{a} = \langle 2, 7, -5 \rangle$ and $\mathbf{b} = \langle 1, 3, 4 \rangle$, then $\mathbf{a} \times \mathbf{b}$ is
(•) $\langle 43, -13, -1 \rangle$ (∗) $\langle 13, 43, -1 \rangle$ (∗) $\langle 43, 13, -1 \rangle$ (∗) $\langle 43, -13, 1 \rangle$ (∗) $\langle 43, -13, -7 \rangle$

**Problem 6:** Evaluate $\oint_C (4y - e^{\sin(x^2)} \cos(3x)\ dx + \big(5x + \arctan(y)\big)\, dy$ where $C$ is the circle $x^2 + y^2 = 4$ with counter-clockwise orientation.
(•) $4\pi$ (∗) $2\pi$ (∗) $16\pi$ (∗) $-36\pi$ (∗) $36\pi$

**Problem 7:** If $\vec{F}(x, y, z) = \langle xz + \sin(100x), xyz - y^2 + y^3, -y^2 + z^2 \rangle$ find $\text{curl}(\vec{F})$.

(•) $\langle -y(z+x), x, yz\rangle$ (∗) $\langle x, -y(z+x), yz\rangle$ (∗) $\langle yz, x, -y(z+x)\rangle$ (∗) $\langle y(z+x), x, yz\rangle$
(∗) $\langle 0,0,0\rangle$

**Problem 8:** Let $\vec{F}(x,y,z) = \langle e^y, xe^y + e^z, ye^z\rangle$ be a conservative vector field. Find a function $f(x,y,z)$ such that $\nabla f = \vec{F}$.
(•) $xe^y + ye^z$ (∗) $(x+y)e^y$ (∗) $xe^{y+z}$ (∗) $xye^{y+z}$ (∗) $(x+y+z)e^{x+z}$

**Problem 9:** Let $\vec{F}(x,y,z) = \left\langle xz + e^y \sin z, xyz + e^{x^2}\cos(z^2+99), -y^2+z^2\right\rangle$. Find $\operatorname{div}(\vec{F})$.
(•) $(x+1)z$ (∗) $xy+z$ (∗) $yz+x$ (∗) $99$ (∗) $3x^2+y$

**Problem 10:** Find the area of the part of the paraboloid $z = x^2+y^2$ that lies under the plane $z = 20$.
(•) $\dfrac{364}{3}\pi$ (∗) $\dfrac{365}{3}\pi$ (∗) $20\pi$ (∗) $365\pi$ (∗) $\dfrac{200}{3}\pi$

**Problem 11:** Let $S = \left\{(x,y,z) \,\middle\|\, x^2+y^2 = 1,\ 0 \leqslant z \leqslant 1+x \right\}$. Use the cylindrical coordinate system to evaluate $\displaystyle\iint_S z\,dS$

(•) $\dfrac{3\pi}{2}$ (∗) $\dfrac{2\pi}{3}$ (∗) $0$ (∗) $2\pi$ (∗) $\dfrac{\pi}{3}$

**Problem 12:** Let $\vec{F} = \langle y, x, z\rangle$ and let $S = \left\{(x,y,z) \,\middle\|\, z = 1-x^2-y^2,\ x^2+y^2 \leqslant 1 \right\}$. A unit normal to $S$ is $\vec{n} = \frac{1}{\sqrt{1+4(x^2+y^2)}}\langle 2x, 2y, 1\rangle$. Find $\displaystyle\iint_S \vec{F} \bullet \vec{n}\,dS$.

(•) $\frac{\pi}{2}$ (∗) $\pi$ (∗) $2\pi$ (∗) $-\pi$ (∗) $0$

**Problem 13:** Let $\vec{F} = \langle -y^2, x, z^2 \rangle$ and let $\vec{r}(t) = \langle \cos t, \sin t, 2 - \sin t \rangle$ for $0 \leqslant t \leqslant 2\pi$. Find $\oint_C \vec{F} \bullet d\vec{r}$.

(•) $\pi$ (∗) $2\pi$ (∗) $-2\pi$ (∗) $-\pi$ (∗) $0$

**Problem 14:** Let $S = \{(x,y,z) \mid x + y + z = 1,\ x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0\}$. A normal vector to $S$ is $\vec{n} = \frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle$. Use Stokes' Theorem to evaluate $\oint_{\partial S} z\,dx - 2x\,dy + 3y\,dz$ orienting $\partial S$ so that if you are standing on $\partial S$ with $S$ on your left, your head points in the direction of $\vec{n}$.

(•) $1$ (∗) $\frac{\sqrt{3}}{2}$ (∗) $\frac{2}{\sqrt{3}}$ (∗) $-\frac{\sqrt{3}}{2}$ (∗) $0$

**Problem 15:** Let $\vec{F}(x,y,z) = \langle z + y^2, 3y + x^3, x + y^2 \rangle$ and let $E = \{(x,y,z) \,\|\, x^2 + y^2 + z^2 \leqslant 1\}$. Use the Divergence Theorem to compute $\oiint_{\partial E} \vec{F} \bullet \vec{n}\,dS$ if the normal to $\partial E$ is the one pointing out of $E$.

(•) $4\pi$ (∗) $2\pi$ (∗) $\frac{2\pi}{3}$ (∗) $-\frac{2\pi}{3}$ (∗) $0$

**Problem 16:** Find the position vector $\vec{r}(t)$ of a particle that has the acceleration vector $\vec{a}(t) = \langle t, e^t, e^{-t} \rangle$, the initial velocity vector $\vec{v}(0) = \langle 0, 1, -1 \rangle$ and the initial position $\vec{r}(0) = \langle 0, 1, 1 \rangle$.

(•) $\left\langle \frac{t^3}{6}, e^t, e^{-t} \right\rangle$ (∗) $\left\langle \frac{t^2}{2}, e^t, -e^{-t} \right\rangle$ (∗) $\langle t, e^t, e^{-t} \rangle$ (∗) $\langle t^3, e^t, e^{-t} \rangle$ (∗) $\left\langle \frac{t^2}{2}, e^t, e^{-t} \right\rangle$

**Problem 17:** Find an equation of the tangent line to the curve $\vec{r}(t) = \langle t, \ln(1-t), e^t \rangle$ at $\vec{r}(0) = \langle 0, 0, 1 \rangle$.
($\bullet$) $x = -y = z - 1$ ($*$) $x = y = z$ ($*$) $x = -y = z$ ($*$) $x = y = \dfrac{z-1}{e}$ ($*$) $x - 1 = y = z - 1$

**Problem 18:** Suppose the temperature at a point $(x, y, z)$ is given by

$$T(x, y, z) = \frac{72}{1 + 6x^2 + 2y^2 + 3z^2}$$

where $T$ is measured in degrees Celsius and $x$, $y$ and $z$ are measured in meters. What is the maximum rate of change in $T$ at the point $(1, 1, 1)$.
($\bullet$) 7 ($*$) 72 ($*$) 6 ($*$) 2 ($*$) 3

**Problem 19:** Find the length of the curve $\vec{r}(t) = \langle 12t, 5\cos t, 5\sin t \rangle$ for $1 \leqslant t \leqslant 2$.
($\bullet$) 26 ($*$) 13 ($*$) 24 ($*$) 17 ($*$) 30

**Problem 20:** Find the shortest distance from $(2, 1, 1)$ to the plane $x + 2y + 2z + 6 = 0$.
($\bullet$) 4 ($*$) 2 ($*$) 6 ($*$) 3 ($*$) 0

**Problem 21:** Let $f(x, y) = x^4 + y^4 - 4xy + 99$, Given that $(-1, -1)$ is a critical point of $f(x, y)$ then it is
($\bullet$) a local minimum ($*$) a saddle point ($*$) an absolute maximum ($*$) can not tell
($*$) a local maximum but not an absolute maximum

**Problem 22:** Evaluate the integral $\int_0^{\sqrt{\pi}} \int_x^{\sqrt{x}} \sin(y^2)\, d\,y\, d\,x$ by reversing the order of integration.

(•) 1 (∗) $\frac{1}{2}$ (∗) $\sqrt{\pi}$ (∗) $\pi$ (∗) 0

**Problem 23:** Use the cylindrical coordinate system to evaluate

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} 5(x^2+y^2)d\,z\,d\,y\,d\,x$$

(•) $16\pi$ (∗) $4\pi$ (∗) $5\pi$ (∗) $3\pi$ (∗) $\pi$

**Problem 24:** Let $x = u^2 - w^2$ and $y = 2uw$. Then $\frac{\partial(x, y)}{\partial(u, w)} =$

(•) $4(u^2 + w^2)$ (∗) $-4(u^2 + w^2)$ (∗) $u^2 + w^2$ (∗) $2(u^2 + w^2)$ (∗) 0

**Problem 25:** Evaluate $\iiint_B 3e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\, d\,V$ using spherical coordinates where $B = \{(x, y, z) \,\|\, x^2+ y^2 + z^2 \leqslant 4\}$.

(•) $4\pi(e^8 - 1)$ (∗) $\frac{4\pi}{3}(e^8 - 1)$ (∗) $\frac{4\pi}{3}(e - 1)$ (∗) $4\pi(e - 1)$ (∗) 1

Answer Key 1

**Math 20550: Calculus** Name:________________________

**Practice Final Exam** *December 2009* Section:________________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 20 multiple choice questions. Please sign the honor statement if you agree:

*"I strictly followed the Notre Dame Honor Code during this test."*

Your Signature ____________________

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| 10. | • | b | c | d | e | 20. | • | b | c | d | e |

1. Let $S$ be the part of cylinder $y^2 + z^2 = 1$, with $z \geq 0$, and $0 \leq x \leq 1$, and let $S$ have the upward orientation. Determine which of the following equals $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle 0, 0, z \rangle$.

(a) $\int_0^1 \int_{-1}^1 \sqrt{1 - y^2} dydx.$

(b) $\int_0^1 \int_{-1}^1 \sqrt{1 - x^2} dydx$

(c) $\int_0^1 \int_{-1}^1 (1 - y^2) dydx$

(d) $\int_0^1 \int_{-1}^1 [\sqrt{1 - y^2}\,]^{-1} dydx$

(e) $\int_0^1 \int_{-1}^1 (1 - x^2) dydx$

2. Find the maximum rate of change of $f(x, y) = x^2y + 2y$ at the point $(-1, 2)$ and the direction in which it occurs.

(a) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle -4, 3 \rangle$.

(b) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle -3, 4 \rangle$.

(c) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle 4, 3 \rangle$.

(d) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle 4, -3 \rangle$.

(e) The maximum rate of change is 6 in the direction $\frac{1}{5}\langle -4, 3 \rangle$.

3. Evaluate $\int_C (1 + x^2y) ds$ where $C$ is the upper half of the unit circle $x^2 + y^2 = 1$.

(a) $\pi + \frac{2}{3}$.

(b) $2\pi$.

(c) 0.

(d) $\frac{2}{3}$.

(e) 1.

4. Determine which of the following integrals gives the volume of the region bounded by the cylinder $x^2 + y^2 = 1$, and the planes $z = 0$ and $x + z = 1$.

(a) $\displaystyle\int_0^{2\pi}\int_0^1\int_0^{1-r\cos\theta} r dz dr d\theta$ (b) $\displaystyle\int_0^{\pi}\int_0^1\int_0^{1-r\cos\theta} r dz dr d\theta$

(c) $\displaystyle\int_0^{2\pi}\int_0^1\int_0^{1-r\cos\theta} dz dr d\theta$ (d) $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{1-r\cos\theta} r dz dr d\theta$

(e) $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^{1-x} dz dx dy$

5. Let $C$ be the curve $\mathbf{r}(t) = \langle t, \cos(2t), 1 + \sin(3t)\rangle$, $0 \le t \le \frac{\pi}{2}$, and let

$$\mathbf{F}(x, y, z) = \langle y(2x + z), x(x + z) - z, y(x - 1) + 2z\rangle.$$

Compute $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$. (Hint: Find $f$ with $\mathbf{F} = \nabla f$).

(a) $-\dfrac{\pi^2}{4}$ (b) 0 (c) $\dfrac{\pi}{2}$ (d) $1 - \dfrac{\pi}{4}$ (e) $\dfrac{\pi^2}{2} - 1$

6. If $z = f(x, y)$, $x = u^2 + v^2$ and $y = u^2 - v^2$, find $\dfrac{\partial^2 z}{\partial u \partial v}$.

(a) $4uv(f_{xx} - f_{yy})$ (b) $2(f_{xx} + f_{yy})$ (c) $4uv(f_{xx} + f_{yy})$

(d) $4uv(f_{xx} + 2f_{yy} - f_{yy})$ (e) $4uv(f_{xx} + 2f_{yy} + f_{yy})$

7. Find the scalar projection, $comp_{\mathbf{v}}(\mathbf{w})$, of the vector $\mathbf{w} = \langle 1, 1, 2\rangle$ onto the vector $\mathbf{v} = \langle 2, -2, 1\rangle$.

(a) $\dfrac{2}{3}$ (b) $-\dfrac{2}{3}$ (c) 1 (d) 2 (e) $\dfrac{2}{\sqrt{6}}$

8. Determine which of the following integrals gives the area of the region in the $xy$-plane below the $x$-axis above $y = x^2 - 2$ and to the left of $y = -2x - 2$.

(a) $\displaystyle\int_{-2}^0\int_{-\sqrt{y+2}}^{-\frac{y}{2}-1} dx dy$ (b) $\displaystyle\int_{-2}^0\int_{-\sqrt{y+2}}^{1-\frac{y}{2}} dx dy$ (c) $\displaystyle\int_{-2}^0\int_{\sqrt{y+2}}^{-\frac{y}{2}-1} dx dy$

(d) $\displaystyle\int_{-\sqrt{2}}^0\int_{-\sqrt{y+2}}^{-\frac{y}{2}-1} dx dy$ (e) $\displaystyle\int_{-2}^0\int_{-2x-2}^{x^2-2} dy dx$

9. Find surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

(a) $\int_0^{2\pi} \int_0^2 r\sqrt{1+4r^2}drd\theta$ (b) $\int_0^{2\pi} \int_0^2 \sqrt{1+4r^2}drd\theta$

(c) $\int_0^{\pi} \int_0^2 r\sqrt{1+4r^2}drd\theta$ (d) $\int_0^{2\pi} \int_0^1 r\sqrt{1+4r^2}drd\theta$

(e) $\int_0^{2\pi} \int_0^4 r\sqrt{1+4r^2}drd\theta$

10. Let $C$ be the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$ oriented counterclockwise. Compute

$$\int_C [\cos x^{100} + x^4y^5]dx + [\sin(e^y) + x^5y^4]dy$$

(a) 0 (b) 1 (c) $-1$ (d) $\frac{4}{5}$ (e) $\frac{5}{4}$

11. Express the area between $x^2 + \frac{y^2}{9} = 1$ and $x^2 + \frac{y^2}{9} = 9$ as an integral, using the substitution $x = r\cos\theta$ and $y = 3r\sin\theta$.

(a) $\int_0^{2\pi} \int_1^3 3rdrd\theta$ (b) $\int_0^{2\pi} \int_1^3 9rdrd\theta$ (c) $\int_0^{2\pi} \int_1^3 3r^2drd\theta$

(d) $\int_0^{2\pi} \int_1^9 3rdrd\theta$ (e) $\int_0^{2\pi} \int_0^3 3rdrd\theta$

12. Calculate the arc length of the helix parameterized by $\mathbf{r} = \langle -3t, 4\cos t, -4\sin t\rangle$ for $0 \leq t \leq \pi$

(a) $5\pi$ (b) 0 (c) $10\pi$ (d) $12\pi$ (e) $2\pi$

13. Find the absolute minimum of $f(x,y) = x^2 + 2y^2 + 4y - 2$ on the disk $x^2 + y^2 \leq 4$.

(a) $-4$ (b) $-6$ (c) 0 (d) $-2$ (e) 14

14. Find a direction vector for the line of intersection of the planes $x + y + 2z = 1$ and $3x - y = 0$.

(a) $\langle 1,3,-2\rangle$ (b) $\langle 1,3,2\rangle$ (c) $\langle 3,-2,1\rangle$ (d) $\langle 3,1,-2\rangle$ (e) $\langle -2,3,1\rangle$

15. Let $C$ be the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 2y + 3$ oriented counter-clockwise with the normal upwards. Use Stokes Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = \langle 2e^y - z, \cos(yz), xe^y \rangle.$$

(a) $2\pi$ (b) $\sqrt{5}\pi$ (c) $\frac{\pi}{\sqrt{5}}$ (d) $\frac{2\pi}{\sqrt{5}}$ (e) 0

16. Evaluate $\int\int\int_E 3e^{[(x^2+y^2+z^2)^{\frac{3}{2}}]} dV$ where $E$ is the upper *half* of the ball radius 1 centered at the origin.

(a) $2\pi(e - 1)$ (b) $\pi(e - 1)$ (c) $4\pi(e - 1)$ (d) $\pi$ (e) $2\pi$

17. Find the minimum of the function $f(x, y, z) = x^2 + y^2 + 2z^2$ on the surface $x^2y^2z = 16$.

(a) 10 (b) 8 (c) 16 (d) 12 (e) 14

18. A particle starts from rest at time $t = 0$ at the origin $(0, 0, 0)$. It then begins to move with acceleration $\mathbf{a}(t) = \langle 1, 6t, 12t^2 \rangle$. Find the time, if ever, at which the particle passes through the point $(1, 1, 1)$.

(a) never (b) $t = 1$ (c) $t = 2$ (d) $t = 3$ (e) $t = 6$

19. Determine two vectors that are tangent to the surface $\mathbf{r}(u, v) = \langle vu^2 - 2u, uv^2 - v, uv \rangle$ at the point $(0, 1, 2)$.

(a) $\langle 2, 1, 1 \rangle, \langle 4, 3, 2 \rangle$ (b) $\langle 2, 4, 2 \rangle, \langle 1, 3, 1 \rangle$ (c) $\langle 0, 1, 1 \rangle, \langle 4, 1, 1 \rangle$

(d) $\langle 1, 2, -1 \rangle, \langle 1, -2, 1 \rangle$ (e) $\langle 0, 2, -1 \rangle, \langle 1, -6, 3 \rangle$

20. Find $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $F(x, y, z) = \langle xy, \frac{3}{4}y, -zy \rangle$ and $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 4$ with the outward orientation.

(a) $8\pi$ (b) $\pi$ (c) $2\pi$ (d) $16\pi$ (e) 0

Answer Key

**Math 20550: Calculus III**

**Practice Final Exam** *Fall, 2008*

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**Math 20550: Calculus III**

**Practice Final Exam** *Fall, 2008*

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| 9. | a | b | c | d | e | 19. | a | b | c | d | e |
| 10. | a | b | c | d | e | 20. | a | b | c | d | e |

1. Let $C$ be the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$ oriented counterclockwise. Compute

$$\int_C [\cos x^{100} + x^4y^5]dx + [\sin(e^y) + x^5y^4]dy$$

(a) $-1$  (b) $1$  (c) $\frac{4}{5}$  (d) $0$  (e) $\frac{5}{4}$

2. Find the maximum rate of change of $f(x,y) = x^2y + 2y$ at the point $(-1,2)$ and the direction in which it occurs.

(a) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle 4,-3\rangle$.

(b) The maximum rate of change is 6 in the direction $\frac{1}{5}\langle -4,3\rangle$.

(c) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle -3,4\rangle$.

(d) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle 4,3\rangle$.

(e) The maximum rate of change is 5 in the direction $\frac{1}{5}\langle -4,3\rangle$.

3. Find a direction vector for the line of intersection of the planes $x + y + 2z = 1$ and $3x - y = 0$.

(a) $\langle 3,1,-2\rangle$  (b) $\langle 3,-2,1\rangle$  (c) $\langle -2,3,1\rangle$  (d) $\langle 1,3,2\rangle$  (e) $\langle 1,3,-2\rangle$

4. Find $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $F(x,y,z) = \langle xy, \frac{3}{4}y, -zy\rangle$ and $S$ is the surface of the sphere $x^2 + y^2 + z^2 = 4$ with the outward orientation.

(a) $8\pi$  (b) $0$  (c) $2\pi$  (d) $16\pi$  (e) $\pi$

5. Determine which of the following integrals gives the area of the region in the $xy$-plane below the $x$-axis above $y = x^2 - 2$ and to the left of $y = -2x - 2$.

(a) $\int_{-2}^{0} \int_{-2x-2}^{x^2-2} dydx$ (b) $\int_{-2}^{0} \int_{-\sqrt{y+2}}^{-\frac{y}{2}-1} dxdy$ (c) $\int_{-2}^{0} \int_{\sqrt{y+2}}^{-\frac{y}{2}-1} dxdy$

(d) $\int_{-2}^{0} \int_{-\sqrt{y+2}}^{1-\frac{y}{2}} dxdy$ (e) $\int_{-\sqrt{2}}^{0} \int_{-\sqrt{y+2}}^{-\frac{y}{2}-1} dxdy$

6. Evaluate $\int_C (1 + x^2 y) ds$ where $C$ is the upper half of the unit circle $x^2 + y^2 = 1$.

(a) $\pi + \frac{2}{3}$. (b) $2\pi$. (c) 0. (d) 1. (e) $\frac{2}{3}$.

7. Calculate the arc length of the helix parameterized by $\mathbf{r} = \langle -3t, 4\cos t, -4\sin t \rangle$ for $0 \leq t \leq \pi$

(a) $5\pi$ (b) $12\pi$ (c) $10\pi$ (d) 0 (e) $2\pi$

8. Determine two vectors that are tangent to the surface $\mathbf{r}(u, v) = \langle vu^2 - 2u, uv^2 - v, uv \rangle$ at the point $(0, 1, 2)$.

(a) $\langle 2, 4, 2 \rangle, \langle 1, 3, 1 \rangle$ (b) $\langle 2, 1, 1 \rangle, \langle 4, 3, 2 \rangle$ (c) $\langle 0, 2, -1 \rangle, \langle 1, -6, 3 \rangle$

(d) $\langle 0, 1, 1 \rangle, \langle 4, 1, 1 \rangle$ (e) $\langle 1, 2, -1 \rangle, \langle 1, -2, 1 \rangle$

9. Express the area between $x^2 + \frac{y^2}{9} = 1$ and $x^2 + \frac{y^2}{9} = 9$ as an integral, using the substitution $x = r\cos\theta$ and $y = 3r\sin\theta$.

(a) $\int_0^{2\pi} \int_1^9 3r dr d\theta$ (b) $\int_0^{2\pi} \int_1^3 9r dr d\theta$ (c) $\int_0^{2\pi} \int_1^3 3r dr d\theta$

(d) $\int_0^{2\pi} \int_0^3 3r dr d\theta$ (e) $\int_0^{2\pi} \int_1^3 3r^2 dr d\theta$

10. Find the absolute minimum of $f(x, y) = x^2 + 2y^2 + 4y - 2$ on the disk $x^2 + y^2 \leq 4$.

(a) $-6$ (b) $-4$ (c) $0$ (d) $-2$ (e) $14$

11. Find surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 4$.

(a) $\int_0^{2\pi} \int_0^4 r\sqrt{1 + 4r^2} dr d\theta$

(b) $\int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2} dr d\theta$

(c) $\int_0^{2\pi} \int_0^1 r\sqrt{1 + 4r^2} dr d\theta$

(d) $\int_0^{\pi} \int_0^2 r\sqrt{1 + 4r^2} dr d\theta$

(e) $\int_0^{2\pi} \int_0^2 r\sqrt{1 + 4r^2} dr d\theta$

12. Evaluate $\iiint_E 3e^{[(x^2+y^2+z^2)^{\frac{3}{2}}]} dV$ where $E$ is the upper *half* of the ball radius 1 centered at the origin.

(a) $2\pi$ (b) $\pi$ (c) $4\pi(e - 1)$ (d) $\pi(e - 1)$ (e) $2\pi(e - 1)$

13. Let $C$ be the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $z = 2y + 3$ oriented counter-clockwise with the normal upwards. Use Stokes Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = \langle 2e^y - z, \cos(yz), xe^y \rangle.$$

(a) $2\pi$ (b) $\frac{2\pi}{\sqrt{5}}$ (c) $\sqrt{5}\pi$ (d) $0$ (e) $\frac{\pi}{\sqrt{5}}$

14. Let $S$ be the part of cylinder $y^2 + z^2 = 1$, with $z \geq 0$, and $0 \leq x \leq 1$, and let $S$ have the upward orientation. Determine which of the following equals $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle 0, 0, z \rangle$.

(a) $\int_0^1 \int_{-1}^1 [\sqrt{1-y^2}\ ]^{-1} dy dx$

(b) $\int_0^1 \int_{-1}^1 \sqrt{1-y^2} dy dx$.

(c) $\int_0^1 \int_{-1}^1 (1-x^2) dy dx$

(d) $\int_0^1 \int_{-1}^1 \sqrt{1-x^2} dy dx$

(e) $\int_0^1 \int_{-1}^1 (1-y^2) dy dx$

15. Let $C$ be the curve $\mathbf{r}(t) = \langle t, \cos(2t), 1 + \sin(3t) \rangle$, $0 \leq t \leq \frac{\pi}{2}$, and let

$$\mathbf{F}(x, y, z) = \langle y(2x+z), x(x+z) - z, y(x-1) + 2z \rangle.$$

Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Hint: Find $f$ with $\mathbf{F} = \nabla f$).

(a) $-\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2} - 1$ (c) $\frac{\pi}{2}$ (d) $0$ (e) $1 - \frac{\pi}{4}$

16. A particle starts from rest at time $t = 0$ at the origin $(0, 0, 0)$. It then begins to move with acceleration $\mathbf{a}(t) = \langle 1, 6t, 12t^2 \rangle$. Find the time, if ever, at which the particle passes through the point $(1, 1, 1)$.

(a) $t = 6$ (b) never (c) $t = 1$ (d) $t = 2$ (e) $t = 3$

17. If $z = f(x, y)$, $x = u^2 + v^2$ and $y = u^2 - v^2$, find $\frac{\partial^2 z}{\partial u \partial v}$.

(a) $4uv(f_{xx} + 2f_{yy} - f_{yy})$ (b) $4uv(f_{xx} - f_{yy})$ (c) $4uv(f_{xx} + 2f_{yy} + f_{yy})$

(d) $4uv(f_{xx} + f_{yy})$ (e) $2(f_{xx} + f_{yy})$

18. Find the scalar projection, $comp_{\mathbf{v}}(\mathbf{w})$, of the vector $\mathbf{w} = \langle 1, 1, 2 \rangle$ onto the vector $\mathbf{v} = \langle 2, -2, 1 \rangle$.

(a) $-\frac{2}{3}$ (b) 1 (c) 2 (d) $\frac{2}{\sqrt{6}}$ (e) $\frac{2}{3}$

19. Find the minimum of the function $f(x, y, z) = x^2 + y^2 + 2z^2$ on the surface $x^2y^2z = 16$.

(a) 8 (b) 16 (c) 14 (d) 12 (e) 10

20. Determine which of the following integrals gives the volume of the region bounded by the cylinder $x^2 + y^2 = 1$, and the planes $z = 0$ and $x + z = 1$.

(a) $\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{1-r\cos\theta} r dz dr d\theta$ (b) $\int_0^{2\pi} \int_0^1 \int_0^{1-r\cos\theta} r dz dr d\theta$

(c) $\int_0^{\pi} \int_0^1 \int_0^{1-r\cos\theta} r dz dr d\theta$ (d) $\int_0^{2\pi} \int_0^1 \int_0^{1-r\cos\theta} dz dr d\theta$

(e) $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{1-x} dz dx dy$