Practice Exam 1
Math 20550
Fall 2010

September 8, 2010

This was the first exam last fall.

1. Find the cosine of the angle between two vectors $\langle 7, 1\rangle$ and $\langle 4, -3\rangle$.

   A. $\frac{1}{\sqrt{2}}$

   B. $\sqrt{2}$

   C. $\frac{1}{2}$

   D. $\frac{\pi}{6}$

   E. $\frac{1}{3}$

2. Find the vector given by the projection of $\mathbf{v} = \langle 3, 1, 4\rangle$ onto $\mathbf{a} = \langle 1, 2, -2\rangle$.

   A. $-\frac{1}{3}\langle 1, 2, -2\rangle$

   B. -1

   C. -3

   D. $\frac{1}{3}\langle 1, 2, -2\rangle$

   E. $-\langle 1, 2, -2\rangle$

3. Find a vector perpendicular to the plane that passes through the three points $P(1, 4, 5)$, $Q(-2, 5, -2)$ and $R(1, -1, 0)$.

   A. $\langle 8, 3, -3\rangle$

   B. $\langle 8, -3, 3\rangle$

   C. $\langle -8, 3, -3\rangle$

   D. $\langle 3, 8, -3\rangle$

   E. $\langle -8, 4, -3\rangle$

4. Find the area of the triangle with the vertices $P(0,0,0)$, $Q(1,2,2)$, and $S(2,1,-2)$.

   A. $\frac{9}{2}$

   B. 9

   C. 3

   D. $-\frac{3}{2}$

   E. 4

5. Let L be a straight line that passes through the points $A(2,4,-3)$ and $B(3,-1,1)$. At what point does this line intersect the yz-plane?

   A. $(0,14,-11)$

   B. $(0,14,11)$

   C. $(0,6,5)$

   D. $(0,-6,5)$

   E. $(0,5,6)$

6. The distance from point $(1,1,1)$ to the plane $2x - y + 2z = 6$ is

   A. 1

   B. -1

   C. 0

   D. 3

   E. 5

7. Find the length of the arc of the curve with vector equation $\mathbf{r}(t) = \langle 5\pi, t^2, \frac{2}{3}t^3 \rangle$ for $0 \le t \le \sqrt{3}$.

   A. $\frac{14}{3}$

   B. 0

   C. $\frac{10}{3}$

   D. 3

   E. $-\frac{10}{3}$

8. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^8+y^7}{x^2+y^2}$

   A. 0

   B. 1

   C. does not exist

   D. 4

   E. 2

9. Find symmetric equations for the tangent line to the helix $\mathbf{r}(t) = \langle 2\cos t, \sin t, t\rangle$ at the point $(0, 1, \frac{\pi}{2})$.

   A. $\frac{x}{-2} = \frac{z-\frac{\pi}{2}}{1}$ and $y = 1$

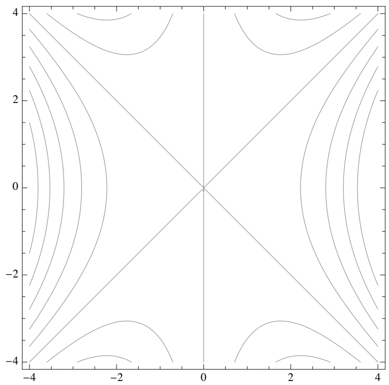
   B. $\frac{x}{2} = \frac{z-\frac{\pi}{2}}{1}$ and $y = 1$

   C. $\frac{x}{-2} = \frac{y-1}{1} = \frac{z-\frac{\pi}{2}}{1}$

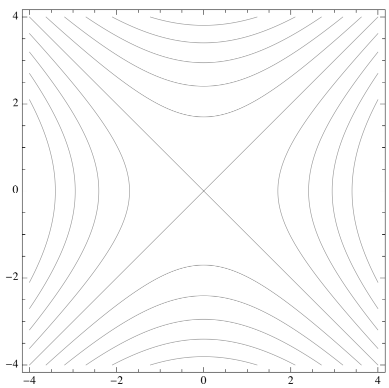
   D. $\frac{x}{2} = \frac{y-1}{1} = \frac{z-\frac{\pi}{2}}{1}$

   E. $\frac{x}{-2} = \frac{z-\frac{\pi}{2}}{1}$ and $y = 0$

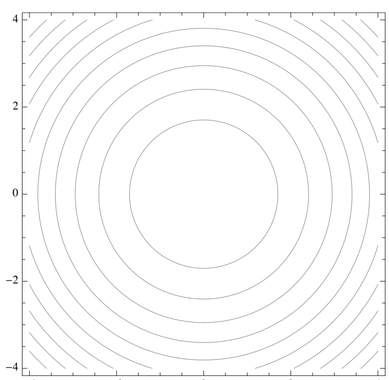
10. Which of the following is the correct contour map for $f(x, y) = x^3 - xy^2 = x(x-y)(x+y)$?

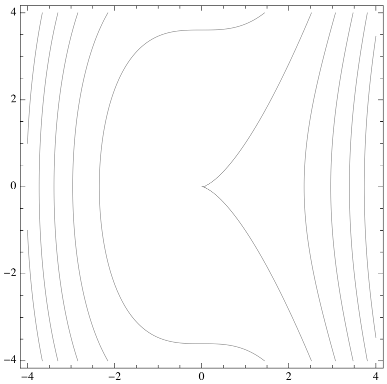
   A.

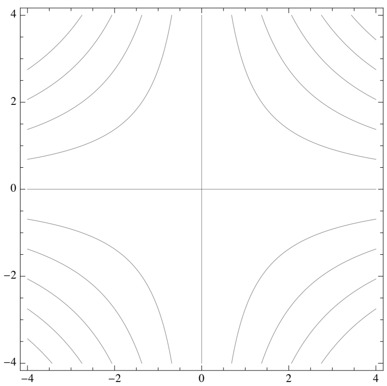
   

   B.

   C.

D. 

E. 

11. (a) Find the velocity vector of a particle that has the given acceleration $\mathbf{a}(t) = \langle 9e^{3t}, -4e^{-2t}, 6t \rangle$ and the given initial velocity $\mathbf{v}(0) = \langle 3, -2, 0 \rangle$.

    (b) If the initial position $\mathbf{r}(0) = \langle 1, -1, 0 \rangle$, then find the position vector $\mathbf{r}(t) =$?

12. Let
$$\mathbf{r}(t) = \langle 12\cos t, 12\sin t, 5t \rangle.$$

    (a) Find $\vec{T}$;

    (b) Find $\vec{N}$;

    (c) Find the tangent component a_T of the acceleration vector of $\mathbf{r}(t)$ at $t = \pi$;

    (d) Find normal component a_N of the acceleration vector of $\mathbf{r}(t)$ at $t = \pi$.

13. Find the osculating plane to $\mathbf{r}(t) = \langle t^2, t^{-1}, t \rangle$ when $t = 1$.

This was an first exam from several years ago. It had 12 multiple choice and 3 partial credit problems.

MATH 20550: Calculus III
Practice Exam 1

Multiple Choice Problems

1. Find an equation for the line through the point $(3, -1, 2)$ and perpendicular to the plane $2x - y + z + 10 = 0$.

   (a) $\frac{x-3}{2} = \frac{y+1}{-1} = z - 2$ (b) $\frac{x+3}{2} = \frac{y-1}{-1} = z - 2$
   (c) $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-2}{2}$ (d) $3x - y + 2z + 10 = 0$
   (e) $3x - 2y + z + 10 = 0$

2. Find an equation of the plane that passes through the point $(1, 2, 3)$ and parallel to $x - y + z = 100$.

   (a) $x-y+z-2 = 0$ (b) $x-y+z+2 = 0$ (c) $x+2y+3z = 100$
   (d) $x - 1 = 2 - y = z - 3$ (e) $x - 1 = \frac{y+1}{2} = \frac{z-1}{3}$.

3. Find the distance between the point $(-1, -1, -1)$ and the plane $x + 2y + 2z - 1 = 0$.

   (a) 2 (b) 0 (c) 6 (d) -2 (e) -6

4. Find values of b such that the vectors $< -11, b, 2 >$ and $< b, b^2, b >$ are orthogonal

   (a) 0, 3, -3 (b) 0, 11, 3 (c) 0, -11, 2
   (d) 0, 2, -2 (a) 0, 11, 2

5. Find the area of the triangle with vertices at the points $(0, 0, 0)$, $(1, 0, -1)$ and $(1, -1, 2)$.

   (a) $\frac{\sqrt{11}}{2}$ (b) $\sqrt{11}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{6}}{2}$ (e) 1

6. Which vector is always orthogonal to $\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}$.

   (a) $\mathbf{a}$ (b) $\mathbf{b}$ (c) $\mathbf{a} - \mathbf{b}$ (d) $|\mathbf{a}|\mathbf{b}$ (e) $proj_{\mathbf{b}}\mathbf{a}$

7. Find the parametric equations of the intersection of the planes $x - z = 0$ and $x - y + 2z + 3 = 0$.

   (a) The line given by $x = -t$, $y = 3 - 3t$ and $z = -t$.
   (b) The line given by $x = -2 - t$, $y = 1 - 3t$ and $z = -t$.
   (c) The line given by $x = 1 + t$, $y = 6 - t$ and $z = 1 + 2t$.
   (d) The plane $3x + 3y - 3z + 3 = 0$.
   (e) The line given by $x = 1 + t$, $y = 6$ and $z = 1 - t$.

8. Find an equation for the normal plane to the vector function

$$\mathbf{r}(t) = \langle e^{t-1}, t^2, \cos(1-t) \rangle$$

   when $t = 1$.

(a) $x + 2y - 3 = 0$.

(b) $x + y + z - 3 = 0$.

(c) $x = 1 + t,\ y = 1 + 2t,\ z = 1$.

(d) Does not exist since $\mathbf{r}'(0) = \mathbf{0}$.

(e) $x + 2y - 3 = 0$ and $z = 0$.

9. The equation of the sphere with center $(4, -1, 3)$ and radius $\sqrt{5}$ is

(a) $(x-4)^2+(y+1)^2+(z-3)^2 = 5$ (b) $(x-4)^2+(y+1)^2+(z-3)^2 = 25$
(c) $(x-4)^2+(y+1)^2+(z-3)^2 = \sqrt{5}$ (d) $(x+4)^2+(y-1)^2+(z+3)^2 = 5$
(e) $(x-4)^2 + (y-1)^2 + (z-3)^2 = 5$

10. Suppose a particle's position at time $t > 0$ is described by

$$\mathbf{r}(t) = < t^2,\ -2t,\ \ln t > .$$

Find the tangential component a_T and normal component a_N of the acceleration at $t = 1$.

(a) $a_T = 1,\ \ a_N = 2$

(b) $a_T = 2,\ \ a_N = 1$

(c) $a_T = 1,\ \ a_N = -2$

(d) $a_T = 1,\ \ a_N = \sqrt{5}$

(e) $a_T = \sqrt{5},\ \ a_N = 1$

11. Find the volume of the parallelepiped determined by the vectors $< 1, 2, 7 >$, $< 0, -3, 4 >$, and $< 0, 0, 6 >$.

(a) 18 (b) 12 (c) 0 (d) 16 (e) 20

12. Skip Problem 12

Partial Credit Problems

13. Find the length of the curve

$$\mathbf{r}(t) = < -\ln t,\ t^2,\ 2t >, \qquad 1 \le t \le e.$$

14. Find the unit normal vector as a function of t, $\mathbf{N}(t)$, where $\mathbf{r}(t) = \langle 3t, 4\cos(t), 4\sin(t)\rangle$.

15. A projectile is fired with initial speed 2 m/s at an elevation angle $\frac{\pi}{6}$ from a height of 4 meters above the ground. Assume the only force acting on the object is gravity. Find the horizontal range (i.e. the horizontal distance travelled before landing) of the projectile. For the simplicity of computation, take the constant acceleration due to gravity $g = 10\ m/s^2$.

Math 20550 Fall 2009

Extra Problems for Practice Exam #1

1. Consider the triangle with vertices $A = (0,1,1)$, $B = (-3,5,1)$, and $C = (1,4,-1)$. Find the angle BAC in radians.

(a) $\cos^{-1}\left(\frac{9}{5\sqrt{14}}\right)$ (b) $\cos^{-1}\left(\frac{3\sqrt{2}}{\sqrt{35}}\right)$ (c) $\cos^{-1}\left(\frac{8\sqrt{2}}{3\sqrt{35}}\right)$ (d) $\cos^{-1}\left(\frac{16}{5\sqrt{21}}\right)$
(e) $\cos^{-1}\left(\frac{5}{7\sqrt{6}}\right)$

2. Let $\mathbf{a} = \langle -1,4,8\rangle$ and $\mathbf{b} = \langle 2,-3,6\rangle$. Compute $\text{comp}_{\mathbf{a}}\mathbf{b}$, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

(a) $34/9$ (b) $34/7$ (c) $34/63$ (d) 62 (e) $\sqrt{62}$

3. Compute the distance from the point $(-1,0,-2)$ to the plane $x - 2y + 3z = 7$.

(a) $\sqrt{14}$ (b) $\sqrt{14}/7$ (c) $\sqrt{68}/39$ (d) $\sqrt{68}$ (e) 0

4. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \langle 2t, -\sin(t), e^t\rangle$ and $\mathbf{r}(0) = \langle 2,-2,2\rangle$.

(a) $\langle t^2+2, \cos(t)-3, e^t+1\rangle$ (b) $\langle 2t^2, \cos(t)-3, e^t+1\rangle$ (c) $\langle t^2+2, -\cos(t)-1, e^t+1\rangle$
(d) $\langle 2t+2, -\sin(t)-2, e^t+1\rangle$ (e) $\langle 2t^2, \cos(t)-3, 2e^t\rangle$

5. Determine which of the following expressions gives the length of the curve defined by $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} + 2\sin t\mathbf{k}$ between the points $(0,1,0)$ and $(2\pi,-1,0)$.

(a) $\int_0^{\pi} \sqrt{4+\sin^2 t + 4\cos^2 t}\, dt$ (b) $\int_0^{\pi} \sqrt{4t^2+\cos^2 t + 4\sin^2 t}\, dt$
(c) $\int_0^{2\pi} \sqrt{4t^2+\cos^2 t + 4\sin^2 t}\, dt$ (d) $\int_0^{2\pi} \sqrt{4+\sin^2 t + 4\cos^2 t}\, dt$
(e) $\int_0^{\pi} (2\mathbf{i} - \sin t\mathbf{j} + 2\cos t\mathbf{k})\, dt$

6. A particle moves with position function $\mathbf{r}(t) = \langle 4\sin t,\ 4\cos t,\ 3t\rangle$. Find the tangential and normal components of acceleration.

(a) $a_T = 0,\ a_N = 4$ (b) $a_T = 4,\ a_N = 3$ (c) $a_T = 4,\ a_N = 0$ (d) $a_T = 0,\ a_N = 0$
(e) $a_T = \frac{4}{5},\ a_N = \frac{4}{5}$

7. Find the equation of the osculating plane for $\mathbf{r}(t) = \langle t^2-1, t^3, t^2+t\rangle$ at the point $(-1,0,0)$.

(a) $y = 0$ (b) $x+1=0$ (c) $z=0$ (d) $x+y+z+1=0$ (e) $x+z+1=0$

8. Find the unit normal vector of the curve $\mathbf{r}(t) =< t, t-1, t^2-2 >$ at the point $(1,0,-1)$.

(a) $\frac{1}{\sqrt{3}} < -1,-1,1 >$ (b) $\frac{1}{\sqrt{5}} < 0,-2,1 >$ (c) $\frac{1}{\sqrt{5}} < -2,0,1 >$ (d) $\frac{1}{\sqrt{2}} < 1,-1,0 >$
(e) $< 0,0,1 >$

9. Find an equation for the intersection of the planes $x-y+2z=2$ and $x+2y+z=-1$.

(a) $x=-5t+1, y=t-1, z=3t$. (b) The planes do not intersect.
(c) $x=-5t+45, y=t-1, z=3t$. (d) $x=5t+1, y=t-1, z=3t$. (e) $-5x+y+3z=2$

10. Find the volume of the box (parallelepiped) determined by position vectors of the following points

$$(3,1,1),\ (1,2,4),\ (-1,5,5).$$

(a) 32 (b) 30 (c) 17 (d) 22 (e) 35

11. Find the radius of the sphere

$$x^2+6x+y^2-4y+z^2-2z=2.$$

(a) 4 (b) $\sqrt{2}$ (c) 3 (d) $2\sqrt{2}$ (e) 2

12. The level curves of the function

$$f(x, y) = \frac{1}{\sqrt{2x - x^2 - y^2}}$$

are

(a) Circles centers at (1, 0) with radius ≤ 1 (b) Circles centers at (1, 0) with arbitrary radius (c) Circles centers at (-1, 0) with radius ≥ 1 (d) straight lines (e) parabolas

13.* Find $\lim_{(x,y)\to(0,0)} \frac{7x^4y^4}{x^2 + y^2}$ if exists

(a) $\frac{7}{2}$ (b) 0 (c) ∞

(d) 7 (e) does not exist

13. Find a *unit* vector that is perpendicular to both of the vectors $\mathbf{a} = \langle -1, 2, 1\rangle$ and $\mathbf{b} = \langle 4, -1, 3\rangle$.
14. Find the point where the line defined by $\mathbf{r}(t) = \langle 2 + t, t, 3 - 3t\rangle$ intersects the plane $3x - y + 2z = 4$.
15. Let C be the curve defined by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and let M be the plane $x - 3y + 3z = 0$.

    (a) Find the points (if any) on C where the *normal plane* to C is parallel to M.

    (b) Find the points (if any) on C where the *tangent line* to C is parallel to M.