MATH 20550: Calculus III
Practice Exam 3

Multiple Choice Problems

1. Find moment about the yz plane of the solid E bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + z = 1$, $x = 0$ and $z = 0$ with density $\rho(x, y, z)$.

   (a) $\int_{-1}^{1}\int_0^{1-y^2}\int_0^{1-z} x\rho(x,y,z)dxdzdy$ (b) $\int_{-1}^{1}\int_0^{1-y^2}\int_0^{1-z} yz\rho(x,y,z)dxdzdy$
   (c) $\int_0^{1}\int_0^{1-y^2}\int_0^{1-z} x\rho(x,y,z)dxdzdy$ (d) $\int_{-1}^{1}\int_0^{1-z}\int_0^{1-y^2} x\rho(x,y,z)dzdxdy$
   (e) $\int_0^{1}\int_0^{1-y^2}\int_0^{1-z} y\rho(x,y,z)dxdzdy$

2. Determine which of the following integrals gives the volume of the solid region bounded by the paraboloid $z = 3y^2 + 3x^2$ and the cone $z = 4 - \sqrt{x^2 + y^2}$.

   (a) $\int_0^{2\pi}\int_0^{1}\int_{3r^2}^{4-r} rdzdrd\theta$ (b) $\int_0^{2\pi}\int_0^{1}\int_0^{4-r} 3r^2dzdrd\theta$
   (c) $\int_0^{2\pi}\int_0^{\pi}\int_{3r^2}^{4-r} r^2 sin\theta drdzd\theta$ (d) $\int_0^{2\pi}\int_0^{3}\int_0^{4-r^2} rdzdrd\theta$
   (e) $\int_0^{2\pi}\int_0^{4}\int_0^{3} rsin\theta drdzd\theta$

3. Evaluate the line integral with respect to arc length

$$\int_C x \, ds$$

   where C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

   (a) $(5\sqrt{5} - 1)/12$ (b) $(2\sqrt{2} - 1)/6$ (c) 0
   (d) $\sqrt{5}/2$ (e) 1

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(t) = 3\sqrt[5]{xy}\,\mathbf{i} - \mathbf{j}$ and C is the curve $y^2 = x^3$ from $(0,0)$ to $(1,1)$.

   (a) 1 (b) 2 (c) 0 (d) -1 (e) 3

5. Find the area inside the cardioid $r = 2 + \cos(\theta)$.

   (a) $\frac{9\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) 6π (d) $\frac{15\pi}{2}$ (e) 3π

6. Consider the loop (one leaf) of the 4-leaf rose $r = \cos 2\theta$ which is entirely contained in the first and fourth quadrant. If this region has density $\rho(x,y) = x^2 + y^2$ then which of the following integrals is the moment about the y-axis.

   (a) $\int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} r^4 \cos\theta \, dr \, d\theta$ (b) $\int_{-\pi/2}^{\pi/2}\int_0^{\cos 2\theta} r^4 \cos\theta \, dr \, d\theta$
   (c) $\int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} r^3 \cos\theta \, dr \, d\theta$ (d) $\int_{-\pi/2}^{\pi/2}\int_0^{\cos 2\theta} r^3 \cos\theta \, dr \, d\theta$
   (e) $\int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} xr^3 \, dr \, d\theta$

7. What is $\int_C (\nabla f) \cdot d\mathbf{r}$ if $f(x, y, z) = xy^2 - xze^{yz}$ and C is a curve from $(1, 0, 1)$ to $(3, 2, 0)$.

   (a) 13 (b) 10 (c) 0 (d) -5 (e) 21

8. Let E be the solid region bounded by $z = x^2 + y^2$ and $z = 3 - 2x^2 - 2y^2$. Suppose the volume of E is $\frac{3\pi}{2}$ and the density of E is constant. Find the center of mass of E.

   (a) $(0, 0, \frac{4}{3})$ (b) $(-\frac{1}{2}, \frac{1}{2}, 3)$ (c) $(0, 0, \sqrt{2})$ (d) $(0, 0, \frac{3}{2})$ (e) $(0, 0, 1)$

Partial Credit Problems

9. Under the change of variables $x = s^2 - t^2$, $y = 2st$, the quarter circular region in the st-plane given by $s^2 + t^2 \leq 1$ is mapped onto a certain region D of the xy-plane. Evaluate

$$\int\int_D \frac{dxdy}{\sqrt{x^2 + y^2}}.$$

10. Let $\mathbf{F} = z^2\mathbf{i} + z \exp yz\mathbf{j} + (2xz + \cos z + y \exp yz)\mathbf{k}$. Find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$.

11. Find the volume of the solid under the surface $z = \sin(x^2 + y^2)$ and above the annulus $D = \{(x, y) \mid \frac{\pi}{4} \leq x^2 + y^2 \leq \frac{\pi}{2}\}$.

12. A lamina of uniform density occupies the region D bounded by the parabola $y = 1 - x^2$ and the x-axis. Find its center of mass.

Math 20550 Fall 2009

Practice Exam III

1. Use a double integral to find the area enclosed by one loop of the rose $r = 2\cos 3\theta$.

(a) $\frac{\pi}{3}$ (b) 4π (c) $\frac{\pi}{2}$ (d) 3π (e) 6π

2. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 4 - x^2 - y^2$.

(a) 4π (b) $\frac{\pi}{2}$ (c) 8π (d) $\frac{\pi}{4}$ (e) 2π

3. The density function of the lamina is given by $\rho(x, y) = 2\sqrt{x^2 + y^2}$ on a semi-circle lamina $\Omega = \{(x, y) \mid x^2 + y^2 = \pi, y \geq 0\}$. Find its center of mass.

(a) $(0, \frac{3}{2})$ (b) $(\frac{3}{2}, 0)$ (c) $(\frac{3}{2\pi}, 0)$ (d) $(0, \frac{3}{2\pi})$ (e) $(3, 3)$

4. Find $\int\int\int_E z dV$, where E is bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 0$, $y = 3x$ and $z = 0$ in the first octant.

(a) $\frac{27}{8}$ (b) 3 (c) 27 (d) 9 (e) 8

5. Evaluate the integral $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dz dx dy$ by changing to cylindrical coordinates.

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{4}{7}$ (d) $\frac{i}{5}$ (e) $\frac{17}{15}$

6. Use a spherical coordinates to evaluate $\iiint_E z dV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

(a) $\frac{15}{16}\pi$ (b) $\frac{16}{15}\pi$ (c) 15π (d) 3π (e) $\frac{4\pi}{3}$

7. Find the volume of the volume of the smaller wedge cut from a sphere of radius 3 by two planes that intersect along a diameter at an angle $\frac{\pi}{6}$.

(a) 3π (b) $\frac{4\pi}{3}$ (c) $\frac{\pi}{9}$ (d) 9π (e) 6π

8. Use the transformation $x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(u - 3v)$ to evaluate the integral $\iint_R (x = 2y) dA$, where R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$ and $(1, 5)$.

(a) 32 (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 64 (e) 16

9. Evaluate $\iiint_E z dV$, where $E = \{(x, y, z) \mid \frac{x^2}{16} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1\}$ by using the transform $x = 4u$, $y = 3v$ and $z = 3w$.

(a) 3π. (b) 6π (c) 9π. (d) 2π. (e) 12π

10. Find the integral $\int_C (x^2y^3 - \sqrt{x}) dy$ where C is the arc of the curve $y = \sqrt{x}$ from the origin $(0, 0)$ to $(4, 2)$.

(a) 15 (b) 16 (c) 8 (d) 4 (e) 2

11. Find the line integral $\int_C xz ds$ where C is the curve $\mathbf{r}(t) = \langle 4\sin t, 3t, 4\cos t \rangle$, $0 \leq t \leq \pi$.

(a) 0 (b) 40 (c) 48 (d) 12 (e) 5

12. Find the integral $\int_C \tan y dx + x(\sec y)^2 dy$ where C is any path from $(0,1)$ to $(2, \frac{\pi}{4})$.

(a) 2 (b) $\frac{\pi}{4}$ (c) 1 (d) 0 (e) $\frac{\pi}{2}$

13. Evaluate the iterated integral $\int_0^3 \int_{-\sqrt{9-y^2}}^0 x^2 y dx dy$ by converting to polar coordinates.

14. Use a triple integral to find the volume of tetrahedron enclosed by the coordinate planes and the plane $x + 2y + z = 4$.

15. Let $\rho(x, y, z) = 1$ be the density function defined on the region E bounded by the paraboloid $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$. Find its center of mass.

Answer Key 1

Calculus III Your Name: ____________________

Exam III *November 27, 2007* Your professor's name: ____________________

Mark an "X" for your answer to each multiple choice question below. For partial credit questions, show all of your work.

Please sign the honor statement if you agree:

"I strictly followed the Notre Dame Honor Code during this test."

Your Signature ____________________

number right times 6 = __________

11.

12.

13.

You start with: 10 points

Total Score __________

1.	a	•	c	d	e	6.	a	b	c	d	•
2.	a	•	c	d	e	7.	a	•	c	d	e
3.	a	b	c	d	•	8.	a	b	c	•	e
4.	a	•	c	d	e	9.	a	•	c	d	e
5.	•	b	c	d	e	10.	a	b	c	•	e

Calculus III Your Name:______________________________

Exam III *November 27, 2007* Your professor's name:__________________

Mark an "X" for your answer to each multiple choice question below. For partial credit questions, show all of your work.

Please sign the honor statement if you agree:

"I strictly followed the Notre Dame Honor Code during this test."

Your Signature ________________________

number right times 6 = ____________

11.

12.

13.

You start with: 10 points

Total Score ____________

1.	a	b	c	d	e	6.	a	b	c	d	e
2.	a	b	c	d	e	7.	a	b	c	d	e
3.	a	b	c	d	e	8.	a	b	c	d	e
4.	a	b	c	d	e	9.	a	b	c	d	e
5.	a	b	c	d	e	10.	a	b	c	d	e

1. Find the double integral $\int\int_D [1+3x+y]dxdy$, where D is a triangle with the vertices $(0,0)$, $(1,0)$ and $(0,2)$.

(a) 2 (b) $\frac{8}{3}$ (c) 3 (d) $\frac{4}{3}$ (e) $\frac{3}{2}$

2. Evaluate $\int_C xyds$, where C is given by $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$ for $0 \le t \le \frac{\pi}{2}$.

(a) 5 (b) 40 (c) 0 (d) 10 (e) -40

3. Find the work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force $\mathbf{F}(x,y,z) = \langle xy, yz, zx\rangle$ in moving a particle along the curve C given by $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$ for $-1 \le t \le 1$.

(a) $\frac{27}{28}$ (b) $\frac{1}{2}$ (c) $\frac{5}{7}$ (d) $\frac{1}{4}$ (e) $\frac{10}{7}$

4. Use a double integral to find the area enclosed by one loop of four-leaved rose $r = \sin 2\theta$, where $D' = \{(r,\theta) \,|\, 0 \le \theta \le \frac{\pi}{2}, 0 \le r \le \sin 2\theta\}$.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{4}$ (d) 0 (e) $-\frac{\pi}{2}$

5. Find the total mass of a lamina D of the density $\rho(x, y) = \sqrt{x^2 + y^2}$, where

$$D = \{(x, y) \mid x^2 + y^2 \leq 4, y \geq 0\}.$$

(a) $\frac{8\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{2}$ (e) $\frac{4}{3}$

6. Use the spherical coordinates to evaluate $\int\int\int_E e^{(x^2+y^2+z^2)^{\frac{3}{2}}} dV$, where

$$E = \{(x, y, z) \mid y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 1\}.$$

(a) 0 (b) $\frac{4\pi}{3}(e-1)$ (c) $\frac{\pi}{3}e$ (d) $4\pi e$ (e) $\frac{\pi}{3}(e-1)$

7. Let $\mathbf{F} = \langle xz, xyz, -y^2 \rangle$. Find $curl\mathbf{F}$.

(a) $\langle x, -y(2+x), yz \rangle$ (b) $\langle -y(2+x), x, yz \rangle$ (c) 0

(d) $z + xy$ (e) $\langle -y(2+x), -x, yz \rangle$

8. Use cylindrical coordinates to evaluate $\int\int\int_E (x^2+y^2)dV$, where

$$E = \{(x,y,z) \mid \sqrt{x^2+y^2} \leq z \leq 2\}.$$

(a) $\frac{4\pi}{9}$ (b) $\frac{3\pi}{4}$ (c) $\frac{4\pi}{3}$ (d) $\frac{16\pi}{5}$ (e) $\frac{\pi}{4}$

9. Use Green's Theorem to evaluate $\oint_C [(3y - e^{x^2})dx + (7x + \sqrt{y^{99}+y+100})dy]$, where C is the circle $x^2+y^2=9$ with the counter-clockwise orientation.

(a) 0 (b) 36π (c) 3π (d) -36π (e) 7π

10. Let $x = 2u$ and $y = -3v$. Then $\int_{-3}^{3}\int_{-2}^{2} f(x,y)dxdy$ can be written as:

(a) $\frac{1}{6}\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$

(b) $-6\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$

(c) $6\int_{-3}^{3}\int_{-2}^{2} f(2u,-3v)dudv$

(d) $6\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$

(e) $-4\int_{-1}^{1}\int_{-1}^{1} f(2u,-3v)dudv$

11. Let $E = \{(x, y, z) \,|\, 0 \leq x + y + z \leq 1, x \geq 0, y \geq 0, z \geq 0\}$ be the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. Find $\int\int\int_E x dV$.

12. Let $\mathbf{F} = \langle y^2 + 1, 2xy + 2y + e^{3z}, 3ye^{3z} + 3z^2 \rangle$.

(a) Find $curl\mathbf{F}$.

(b) Find a function f such that $\nabla f = \mathbf{F}$ if such a function exists.

(c) Verify that, for your f, the equation $\nabla f = \mathbf{F}$ holds. (In all parts, be sure to show all work.)

13. Is there a vector field **G** on the whole space such that $curl\mathbf{G} = \langle 2yz, xyz, 3xy \rangle$? Explain why. (No explanation, no credit).