Math 20550 Fall 2009

Extra Problems for Practice Exam #1

1. Consider the triangle with vertices $A = (0,1,1)$, $B = (-3,5,1)$, and $C = (1,4,-1)$. Find the angle BAC in radians.

(a) $\cos^{-1}\left(\frac{9}{5\sqrt{14}}\right)$ (b) $\cos^{-1}\left(\frac{3\sqrt{2}}{\sqrt{35}}\right)$ (c) $\cos^{-1}\left(\frac{8\sqrt{2}}{3\sqrt{35}}\right)$ (d) $\cos^{-1}\left(\frac{16}{5\sqrt{21}}\right)$
(e) $\cos^{-1}\left(\frac{5}{7\sqrt{6}}\right)$

2. Let $\mathbf{a} = \langle -1, 4, 8\rangle$ and $\mathbf{b} = \langle 2, -3, 6\rangle$. Compute $\text{comp}_{\mathbf{a}}\mathbf{b}$, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

(a) $34/9$ (b) $34/7$ (c) $34/63$ (d) 62 (e) $\sqrt{62}$

3. Compute the distance from the point $(-1, 0, -2)$ to the plane $x - 2y + 3z = 7$.

(a) $\sqrt{14}$ (b) $\sqrt{14}/7$ (c) $\sqrt{68}/39$ (d) $\sqrt{68}$ (e) 0

4. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \langle 2t, -\sin(t), e^t\rangle$ and $\mathbf{r}(0) = \langle 2, -2, 2\rangle$.

(a) $\langle t^2 + 2, \cos(t) - 3, e^t + 1\rangle$ (b) $\langle 2t^2, \cos(t) - 3, e^t + 1\rangle$ (c) $\langle t^2 + 2, -\cos(t) - 1, e^t + 1\rangle$
(d) $\langle 2t + 2, -\sin(t) - 2, e^t + 1\rangle$ (e) $\langle 2t^2, \cos(t) - 3, 2e^t\rangle$

5. Determine which of the following expressions gives the length of the curve defined by $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} + 2\sin t\mathbf{k}$ between the points $(0,1,0)$ and $(2\pi, -1, 0)$.

(a) $\int_0^{\pi} \sqrt{4 + \sin^2 t + 4\cos^2 t}\, dt$ (b) $\int_0^{\pi} \sqrt{4t^2 + \cos^2 t + 4\sin^2 t}\, dt$
(c) $\int_0^{2\pi} \sqrt{4t^2 + \cos^2 t + 4\sin^2 t}\, dt$ (d) $\int_0^{2\pi} \sqrt{4 + \sin^2 t + 4\cos^2 t}\, dt$
(e) $\int_0^{\pi} (2\mathbf{i} - \sin t\mathbf{j} + 2\cos t\mathbf{k})\, dt$

6. A particle moves with position function $\mathbf{r}(t) = \langle 4\sin t,\ 4\cos t,\ 3t\rangle$. Find the tangential and normal components of acceleration.

(a) $a_T = 0,\ a_N = 4$ (b) $a_T = 4,\ a_N = 3$ (c) $a_T = 4,\ a_N = 0$ (d) $a_T = 0,\ a_N = 0$
(e) $a_T = \frac{4}{5},\ a_N = \frac{4}{5}$

7. Find the equation of the osculating plane for $\mathbf{r}(t) = \langle t^2-1, t^3, t^2+t\rangle$ at the point $(-1,0,0)$.

(a) $y = 0$ (b) $x+1 = 0$ (c) $z = 0$ (d) $x+y+z+1 = 0$ (e) $x+z+1 = 0$

8. Find the unit normal vector of the curve $\mathbf{r}(t) =< t, t-1, t^2-2 >$ at the point $(1,0,-1)$.

(a) $\frac{1}{\sqrt{3}} < -1,-1,1 >$ (b) $\frac{1}{\sqrt{5}} < 0,-2,1 >$ (c) $\frac{1}{\sqrt{5}} < -2,0,1 >$ (d) $\frac{1}{\sqrt{2}} < 1,-1,0 >$
(e) $< 0,0,1 >$

9. Find an equation for the intersection of the planes $x-y+2z = 2$ and $x+2y+z = -1$.

(a) $x = -5t+1, y = t-1, z = 3t$. (b) The planes do not intersect.
(c) $x = -5t+45, y = t-1, z = 3t$. (d) $x = 5t+1, y = t-1, z = 3t$. (e) $-5x+y+3z = 2$

10. Find the volume of the box (parallelepiped) determined by position vectors of the following points

$$(3,1,1),\ (1,2,4),\ (-1,5,5).$$

(a) 32 (b) 30 (c) 17 (d) 22 (e) 35

11. Find the radius of the sphere

$$x^2+6x+y^2-4y+z^2-2z = 2.$$

(a) 4 (b) $\sqrt{2}$ (c) 3 (d) $2\sqrt{2}$ (e) 2

12. The level curves of the function

$$f(x,y) = \frac{1}{\sqrt{2x - x^2 - y^2}}$$

are

(a) Circles centers at (1, 0) with radius ≤ 1 (b) Circles centers at (1, 0) with arbitrary radius (c) Circles centers at (-1, 0) with radius ≥ 1 (d) straight lines (e) parabolas

13.* Find $\lim_{(x,y)\to(0,0)} \frac{7x^4y^4}{x^2+y^2}$ if exists

(a) $\frac{7}{2}$

(b) 0

(c) ∞

(d) 7

(e) does not exist

13. Find a *unit* vector that is perpendicular to both of the vectors $\mathbf{a} = \langle -1, 2, 1 \rangle$ and $\mathbf{b} = \langle 4, -1, 3 \rangle$.
14. Find the point where the line defined by $\mathbf{r}(t) = \langle 2 + t, t, 3 - 3t \rangle$ intersects the plane $3x - y + 2z = 4$.
15. Let C be the curve defined by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ and let M be the plane $x - 3y + 3z = 0$.

    (a) Find the points (if any) on C where the *normal plane* to C is parallel to M.

    (b) Find the points (if any) on C where the *tangent line* to C is parallel to M.