Math 20550 Practics Exam 2 Fall 2009

1. Find f_{yy} for $f(x,y) = \int_y^x e^{-t^2}\,dt$.

(a) $2ye^{-y^2}$ (b) 0 (c) $-2ye^{-y^2}$ (d) ye^{-y^2} (e) $-ye^{-y^2}$ I

2. Let $H = xe^{y-z^2}$, $x = 2uv$, $y = u - v$ and $z = u + v$. Find $\dfrac{\partial H}{\partial u}$ when $u = 3$ and $v = -1$.

(a) 16 (b) 36 (c) 3 (d) -1 (e) 2

3. Let $f(x,y,z) = x\sin(yz)$. Find the directional derivative at $(1,3,0)$ in the direction $\vec{v} = \langle 1, 2, -2\rangle$.

(a) -2 (b) 2 (c) 1 (d) 3 (e) -3

A4. Find the maximum rate of change of $f(x,y) = \sin(xy)$ at $(1,0)$.

(a) 1 (b) -1 (c) 0 (d) $\cos 1$ (e) $\sin 1$

B4. Find the maximum volume of a rectangular box such that the sum of lengths of its 12 edges is 24.

(a) 8 (b) 12 (c) $(12)^3$ (d) 1 (e) 0

A5. Suppose that $(1,-1)$ is a critical point of a smooth function $f(x,y)$ with continuous second derivatives, $f_{xx}(-1,1) = 3$, $f_{xy}(-1,1) = 2$ and $f_{yy}(-1,1) = 2$. What can you say about the type of the critical point of $(-1,1)$ of f?

(a) a local minimum (b) a saddle point (c) a local maximum

(d) no information (e) absolute maximum

B5. Find the maximum value of $f(x,y,z) = xyz$ subject to $x^2 + 2y^2 + 3z^2 = 6$.

(a) $\frac{2}{\sqrt{3}}$ (b) 1 (c) $-\frac{2}{\sqrt{3}}$ (d) 0 (e) 6

A6. Find an equation of the tangent plane at the point $(3,-1,2)$ to the ellipsoid

$$\frac{x^2}{9} + y^2 + \frac{z^2}{4} = 3.$$

(a) $\frac{2}{3}x - 2y + z - 6 = 0$ (b) $\frac{2}{3}x + 2y + z - 6 = 0$

(c) $\frac{2}{3}x - 2y + z + 6 = 0$ (d) $\frac{3}{x} - y + 2z - 6 = 0$

(e) $\frac{3(x-3)}{2} = \frac{y+1}{-2} = z - 2$

B 6. Find the volume of the solid bounded by the surface $z = 6 - xy$ and the plane $x = 2$, $x = -2$, $y = 0$, $y = 3$ and $z = 0$.

(a) 72 (b) 36 (c) 6 (d) 3 (e) 0

A7. Find the volume of the solid that lies under hyperbolic paraboloid $z = 4 + x^2 - y^2$ and above the square $R = [-1,1] \times [0,2] = \{(x,y) \mid -1 \le x \le 1, 0 \le y \le 2\}$.

(a) 12 (b) 4 (c) 3 (d) 2 (e) 1

B7. Find $\displaystyle\iint_D \frac{2y}{x^2+1}\,dA$ where $D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$.

(a) $\frac{1}{2}\ln 2$ (b) $\ln 2$ (c) 1 (d) 0 (e) $-\frac{1}{2}\ln 2$

A8. If $D = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$, find the integral $\int\int_D \sqrt{1 - y^2}\,dxdy$. (Hint: not to use polar coordinates).

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{2}$ (e) $\frac{\pi}{3}$

B8. Find $\int\int_D 2xy\,dA$, where D is the triangular region with vertices $(0, 0)$, $(1, 2)$ and $(0, 3)$.

(a) $\frac{7}{4}$ (b) $\frac{4}{7}$ (c) 2 (d) 3 (e) 0

9. Evaluate the integral by reversing the order of integration $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1}\,dxdy$

(a) $\frac{52}{9}$ (b) 4 (c) $\frac{26}{9}$ (d) $\frac{26}{3}$ (e) 2

10. Find $\frac{\partial z}{\partial x}$ if $x^4 + y^4 + z^4 + 4xyz = 100$.

(a) $-\frac{x^3 + yz}{z^3 + xy}$ (b) $\frac{x^3 + yz}{z^3 + xy}$ (c) $-\frac{z^3 + xy}{x^3 + yz}$ (d) $-\frac{z^3 + xy}{x^3 + yz}$ (e) $100y$

Solutions to all mutiple choice problems are (a).

Partial Credit Problems

11. (i) Find the points on the sphere $x^2 + y^2 + z^2 = 9$ where the tangent plane is parallel to the plane $2x - y + 2z = 99$.

(ii) Find equations of normal lines to the sphere $x^2 + y^2 + z^2 = 9$ at points derived in part (i) above.

12. Find the absolute maximum and absolute minimum values of $f(x, y) = 2x - y$ on the domain $D = \{(x, y) \mid x^2 + \frac{y^2}{4} \leq 2\}$.

13. Let E be the largest rectangle box with edges parallel to axes that can be inscribed in the ellipsoid $9x^2 + 36y^2 + 4z^2 = 108$. Find the volume of E. (Hint: the box intersects all octants.)

14. Find the maximum and minimum values of $f(x, y, z) = yz + xy$ subject to constraints $xy = 1$ and $y^2 + z^2 = 1$.

15. Evaluate the integral $\int_0^8 \int_{y^{\frac{1}{3}}}^2 e^{x^4}\, dx dy$.