Math 20550 Fall 2009

Practice Exam III

1. Use a double integral to find the area enclosed by one loop of the rose $r = 2\cos 3\theta$.

(a) $\frac{\pi}{3}$ (b) 4π (c) $\frac{\pi}{2}$ (d) 3π (e) 6π

2. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 4 - x^2 - y^2$.

(a) 4π (b) $\frac{\pi}{2}$ (c) 8π (d) $\frac{\pi}{4}$ (e) 2π

3. The density function of the lamina is given by $\rho(x, y) = 2\sqrt{x^2 + y^2}$ on a semi-circle lamina $\Omega = \{(x, y) \mid x^2 + y^2 = \pi, y \geq 0\}$. Find its center of mass.

(a) $(0, \frac{3}{2})$ (b) $(\frac{3}{2}, 0)$ (c) $(\frac{3}{2\pi}, 0)$ (d) $(0, \frac{3}{2\pi})$ (e) $(3, 3)$

4. Find $\int\int\int_E z dV$, where E is bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 0$, $y = 3x$ and $z = 0$ in the first octant.

(a) $\frac{27}{8}$ (b) 3 (c) 27 (d) 9 (e) 8

5. Evaluate the integral $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dz dx dy$ by changing to cylindrical coordinates.

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{4}{7}$ (d) $\frac{i}{5}$ (e) $\frac{17}{15}$

6. Use a spherical coordinates to evaluate $\int\int\int_E zdV$, where E lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant.

(a) $\frac{15}{16}\pi$ (b) $\frac{16}{15}\pi$ (c) 15π (d) 3π (e) $\frac{4\pi}{3}$

7. Find the volume of the volume of the smaller wedge cut from a sphere of radius 3 by two planes that intersect along a diameter at an angle $\frac{\pi}{6}$.

(a) 3π (b) $\frac{4\pi}{3}$ (c) $\frac{\pi}{9}$ (d) 9π (e) 6π

8. Use the transformation $x=\frac{1}{4}(u+v)$, $y=\frac{1}{4}(u-3v)$ to evaluate the integral $\int\int_R (x=2y)dA$, where R is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1)$ and $(1,5)$.

(a) 32 (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 64 (e) 16

9. Evaluate $\int\int\int_E zdV$, where $E=\{(x,y,z)\,|\,\frac{x^2}{16}+\frac{y^2}{4}+\frac{z^2}{9}\leq 1\}$ by using the transform $x=4u$, $y=3v$ and $z=3w$.

(a) 3π. (b) 6π (c) 9π. (d) 2π. (e) 12π

10. Find the integral $\int_C (x^2y^3-\sqrt{x})dy$ where C is the arc of the curve $y=\sqrt{x}$ from the origin $(0,0)$ to $(4,2)$.

(a) 15 (b) 16 (c) 8 (d) 4 (e) 2

11. Find the line integral $\int_C xzds$ where C is the curve $\mathbf{r}(t)=\langle 4\sin t, 3t, 4\cos t\rangle$, $0\leq t\leq\pi$.

(a) 0 (b) 40 (c) 48 (d) 12 (e) 5

12. Find the integral $\int_C \tan y dx + x(\sec y)^2 dy$ where C is any path from $(0, 1)$ to $(2, \frac{\pi}{4})$.

(a) 2 (b) $\frac{\pi}{4}$ (c) 1 (d) 0 (e) $\frac{\pi}{2}$

13. Evaluate the iterated integral $\int_0^3 \int_{-\sqrt{9-y^2}}^0 x^2 y dx dy$ by converting to polar coordinates.

14. Use a triple integral to find the volume of tetrahedron enclosed by the coordinate planes and the plane $x + 2y + z = 4$.

15. Let $\rho(x, y, z) = 1$ be the density function defined on the region E bounded by the paraboloid $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$. Find its center of mass.

Corrections to the practice exam for exam III:

- Number 1 is correct as written, and the answer is (a).

  [Note: when this question was on last year's exam, students were provided with the double-angle formula $\cos 2A = 2\cos^2 A - 1$.]

- The answer to number 2 is (c): 8π.

- Numbers 3 - 6 are correct as written, and their answers are all (a).

- Number 7 contains a typo: "the volume of" is written twice; otherwise, it's fine, and the answer is (a).

- Number 8 contains a typo: the function inside the integral is listed as "$x = 2y$". It should be "$x - 2y$". The answer is -16, which is not listed as one of the choices.

  [Note: even if you assume the function should be $x + 2y$, the answer you get is still not one of the listed choices.]

- Number 9 contains a typo: the transformation should include $y = 2v$, NOT $y = 3v$. The answer is 0, which is not listed as one of the choices.

- The answer to number 10 is 30, which is not listed as one of the choices.

- Numbers 11 and 12 are correct as written, and their answers are (a).