

Name: _____

Instructor: _____

Math 10360, Exam 3

April 22, 2008

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)
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Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

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Multiple Choice

1.(5 pts.) Which of the following is TRUE ?

(a) $\int_2^{\infty} dx$ is convergent.

(b) $\int_2^{\infty} \frac{1}{\ln x} dx$ is convergent.

(c) $\int_2^{\infty} \ln x dx$ is convergent.

(d) $\int_2^{\infty} \frac{1}{x^p} dx$ is convergent for $p \geq 1$.

(e) $\int_2^{\infty} \frac{1}{x^p} dx$ is convergent for $p > 1$.

2.(5 pts.) Evaluate the series

$$\sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right) =$$

- (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ (e) divergent

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3.(5 pts.) Determine if the series

$$\sum_{n=0}^{\infty} \frac{e}{4^n}$$

is convergent or divergent. Find the sum if it is convergent.

- (a) $\frac{4}{3}e$ (b) $e/4$ (c) $\frac{3}{4}e$ (d) $\frac{e}{1 - \frac{e}{4}}$ (e) divergent

4.(5 pts.) The sequence $\{a_n\}$ is defined recursively by the formulas

$$a_1 = 1, \quad a_{k+1} = (k+1)a_k.$$

Find the first four terms of the sequence.

- (a) 1, 3, 5, 7
(b) 1, 2, 3, 4
(c) 1, 2, 4, 8
(d) 1, 2, 6, 24
(e) 1, 2, 8, 16

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5.(5 pts.) Which of the following statements is TRUE?

- (a) The infinite series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$ is convergent.
- (b) The infinite series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ is convergent.
- (c) The infinite series $\sum_{n=1}^{\infty} \left(\sin \frac{\pi}{3}\right)^n$ is convergent.
- (d) The infinite series $\sum_{n=1}^{\infty} (-1)^n$ is convergent.
- (e) The infinite series $\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$ is convergent.

6.(5 pts.) Determine whether the following integral is convergent or divergent

$$\int_0^{\infty} e^{-x} dx.$$

Evaluate the integral if it is convergent.

- (a) -1 (b) divergent (c) e (d) 1 (e) $-e$

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7.(5 pts.) The partial fraction decomposition of the function

$$\frac{x + 1}{(x - 1)^2(x^2 + 3x - 4)}$$

is of the form:

(a) $\frac{Ax + B}{x^2 + 3x - 4} + \frac{C}{(x - 1)^2}$

(b) $\frac{A}{x + 4} + \frac{Bx + C}{(x - 1)^3}$

(c) $\frac{A}{x + 4} + \frac{B}{x - 1} + \frac{Cx + D}{(x - 1)^2}$

(d) $\frac{A}{x^2 + 3x - 4} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2}$

(e) $\frac{A}{x + 4} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3}$

8.(5 pts.) Determine whether the limit

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$$

exists. Evaluate the limit if it exists.

- (a) DNE (b) 0 (c) 2 (d) 1/2 (e) $\ln 2$

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9.(5 pts.) Evaluate the indefinite integral

$$\int \sin^3 x \, dx.$$

(a) $\sin x - \frac{1}{3} \sin^3 x + C$ (b) $\frac{1}{3} \sin^3 x - \sin x + C$ (c) $\frac{1}{4} \sin^4 x + C$

(d) $\cos x - \frac{1}{3} \cos^3 x + C$ (e) $\frac{1}{3} \cos^3 x - \cos x + C$

10.(5 pts.) Evaluate the indefinite integral

$$\int \sec^4 x \, dx.$$

(Hint: $\sec^2 x = 1 + \tan^2 x$)

(a) $\frac{1}{3} \tan^3 x + \tan x + C$ (b) $\frac{1}{3} \sec^3 x - \sec x + C$ (c) $\tan x - \frac{1}{3} \tan^3 x + C$

(d) $\frac{1}{5} \sec^5 x + C$ (e) $\sec x + \frac{1}{3} \sec^3 x + C$

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11.(5 pts.) Evaluate the limit

$$\lim_{x \rightarrow 0^+} (1 + 2x)^{1/x}.$$

(a) e

(b) e^2

(c) $e^{1/2}$

(d) 2

(e) $2e$

12.(5 pts.) Find the second Taylor polynomial of the function $f(x) = \ln x$ at $c = 1$.

(a) $1 + x + \frac{1}{2}x^2$

(b) $(x - 1) + \frac{1}{2}(x - 1)^2$

(c) $(x - 1) - \frac{1}{2}(x - 1)^2$

(d) $1 + (x - 1) - \frac{1}{2}(x - 1)^2$

(e) $1 + (x - 1) + \frac{1}{2}(x - 1)^2$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(10 pts.) Follow the instructions in each parts below.

(a) Express the function

$$\frac{x + 1}{x(1 + x^2)}$$

as a partial fraction.

(b) Evaluate each of the following integrals:

$$\int \frac{1}{x} dx =$$

$$\int \frac{1}{1 + x^2} dx =$$

$$\int \frac{x}{1 + x^2} dx =$$

(c) Evaluate the integral:

$$\int \frac{x + 1}{x(1 + x^2)} dx =$$

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14.(10 pts.) Evaluate the following improper integrals.

(a) Express the improper integral below as a limit.

$$\int_1^e \frac{1}{x\sqrt{\ln x}} dx.$$

(b) Determine whether the integral in (a) is convergent or divergent. Show clearly all your calculations.

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15.(10 pts.) Determine whether the following series or series are convergent or divergent. **Indicate clearly how you arrive at the conclusion.**

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n}$.

(b) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$.

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16.(10 pts.) Determine whether the following series are convergent or divergent. State clearly the test(s) that you used in each part. **Indicate clearly how you arrive at the conclusion.**

(a)
$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{5n^6 + 2n + 100}}.$$

(b)
$$\sum_{n=1}^{\infty} \frac{2^n}{(3n)!}.$$

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