Math 10360 Name: ______________________________

Exam 3 Practice Instructor: ______________________________

This is a practice exam for exam 3. Note that the answers provided are explanatory and thus provide more explanation but less computation than you are expected to show on your exam.

Section 1. Short Answer Questions

Supposing the sequences a_n, c_n have the following limits $\lim_{n\to\infty} a_n = 3$, $\lim_{n\to\infty} c_n = 2$ when answering the next three problems.

1. What is $\lim_{n\to\infty} a_n \cdot c_n - a_n$?
2. Suppose the sequence b_n satisfies the following inequalities for every n, $c_n + 1 \leq b_n \leq a_n$. What can be said about $\lim_{n\to\infty} b_n$.
3. What is $\lim_{n\to\infty} a_{n+1} - a_n$?
4. What is $\lim_{n\to\infty} 2^{c_n}$?
5. Compute the indefinite integral $\int \frac{2x}{\sqrt{1-x^4}} \mathrm{d}x$.
6. Compute the indefinite integrals

$$\int \ln(x) \mathrm{d}x$$
$$\int \sin(3x)\cos(x) \mathrm{d}x$$

7. Compute the indefinite integral $\frac{1}{x^3 - x} \mathrm{d}x$.
8. Compute $\int_1^\infty \frac{1}{x \ln(x)} \mathrm{d}x$.
9. Evaluate $\lim_{x\to 0} (\sin(x))^{\sin(x)}$.

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Answer Key for Exam A

Section 1. Short Answer Questions

Supposing the sequences a_n, c_n have the following limits $\lim_{n\to\infty} a_n = 3$, $\lim_{n\to\infty} c_n = 2$ when answering the next three problems.

1. What is $\lim_{n\to\infty} a_n \cdot c_n - a_n$?

   Answer: Since both sequences converge by a result from the textbook

   $$\lim_{n\to\infty} a_n \cdot c_n - a_n = \left(\lim_{n\to\infty} a_n\right) \cdot \left(\lim_{n\to\infty} c_n\right) - \left(\lim_{n\to\infty} a_n\right) = 3$$

2. Suppose the sequence b_n satisfies the following inequalities for every n, $c_n + 1 \leq b_n \leq a_n$. What can be said about $\lim_{n\to\infty} b_n$.

   Answer: By the same reasoning as above $\{c_n + 1\}$ is a convergent sequence with limit 3 so by the squeeze theorem $\lim_{n\to\infty} b_n$ converges to the value 3.

3. What is $\lim_{n\to\infty} a_{n+1} - a_n$?

   Answer: Both the sequence $\{a_n\}$ and $\{a_{n+1}\}$ have the same limit so the reasoning from problem 1 the answer is $3 - 3 = 0$

4. What is $\lim_{n\to\infty} 2^{c_n}$?

   Answer: Since $\lim_{n\to\infty} c_n = 2$ and 2^x is continuous $\lim_{n\to\infty} 2^{c_n} = 2^{\lim_{n\to\infty} c_n} = 4$

5. Compute the indefinite integral $\int \frac{2x}{\sqrt{1-x^4}} \mathrm{d}x$.

   Answer: Use the trig substitution $x^2 = \sin(u)$ and differentiate to get $2x\mathrm{d}x = \cos(u)\mathrm{d}u$. This yields the integral $\int \frac{\cos(u)}{\sqrt{1-\sin^2(u)}}$. Simplifying we discover that the $\cos(u)$ term cancels leaving us with the idefinite integral of $u + C$ and substituting back we get $\arcsin(x^2) + C$ as the answer.

6. Compute the indefinite integrals

$$\int \ln(x)\mathrm{d}x$$
$$\int \sin(3x)\cos(x)\mathrm{d}x$$

Answer: Both integrals can be evaluated using parts. In the first case we let $u = \ln(x)$ and $\mathrm{d}v = 1\mathrm{d}x$ to arrive at the answer $x\ln(x) - x + C$.
The second integral can be computed by applying parts twice and then solving as follows:

$$\begin{aligned}\int \sin(3x)\cos(x)\mathrm{d}x &= \sin(3x)\sin(x) - \int 3\cos(3x)\sin(x)\mathrm{d}x \\ &= \sin(3x)\sin(x) + 3\cos(3x)\cos(x) + \int 9\sin(3x)\cos(x) \\ -8\cdot\int \sin(3x)\cos(x)\mathrm{d}x &= \sin(3x)\sin(x) + 3\cos(3x)\cos(x) \\ \int \sin(3x)\cos(x)\mathrm{d}x &= \frac{\sin(3x)\sin(x) + 3\cos(3x)\cos(x)}{-8}\end{aligned}$$

7. Compute the indefinite integral $\frac{1}{x^3 - x}\mathrm{d}x$.

Answer: We compute this by partial fractions.

$$\begin{aligned}\frac{2}{x^3 - x} &= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \\ \frac{2}{x^3 - x} &= \frac{-2}{x} + \frac{1}{x+1} + \frac{1}{x-1} \\ \int \frac{2}{x^3 - x} &= \int \frac{-2}{x} + \frac{1}{x+1} + \frac{1}{x-1}\mathrm{d}x \\ \int \frac{2}{x^3 - x} &= -2\ln(x) + \ln(x+1) + \ln(x-1) + C\end{aligned}$$

8. Compute $\int_1^\infty \frac{1}{x\ln(x)}\mathrm{d}x$.

Answer:

$$\int_1^\infty \frac{1}{x\ln(x)}\mathrm{d}x = \lim_{b\to 1^+}\int_b^2 \frac{1}{x\ln(x)}\mathrm{d}x + \lim_{c\to\infty}\int_2^c \frac{1}{x\ln(x)}\mathrm{d}x \tag{1}$$

Using $u = \ln(x)$ we can see that the indefinite integral of $\int \frac{1}{x\ln(x)}\mathrm{d}x$ is $\ln(\ln(x)) + C$. Noticing that one of the above limits diverges we conclude the integral diverges (both limits diverge but it's enough that one of them diverge to cause the integral to diverge).

9. Evaluate $\lim_{x\to 0}(\sin(x))^{\sin(x)}$.

Answer:

$$\lim_{x\to 0}(\sin(x))^{\sin(x)} = e^{\lim_{x\to 0}\frac{\ln(\sin(x))}{\frac{1}{\sin(x)}}} \tag{2}$$

Using L'Hopital's rule we see:

$$\lim_{x\to 0}\frac{\ln(\sin(x))}{\frac{1}{\sin(x)}} = \lim_{x\to 0} -\sin(x) = 0 \tag{3}$$

Thus giving us the answer of 1