Calculus 1a
Quiz 3, Section: 106

Name:________________________

Section 1. Short Answer (Show Your Work)

1. Find $\lim_{x \to -\infty} 2x + \sqrt{4x^2 + 2x}$

2. Find $\lim_{x \to 0} \frac{\tan(\sin(x))}{x}$
   Hint: Try multiplying by something clever before substituting.

3. Suppose $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to \infty} g(x) = 2$, $\lim_{x \to \infty} h(x) = 0$

   Calculate: $\lim_{x \to \infty} g(x) + \dfrac{1}{f(x) + h(x)}$

Answer Key for Section 106: Quiz 3

Section 1. Short Answer (Show Your Work)

1. Find $\lim_{x\to-\infty} 2x+\sqrt{4x^2+2x}$

$$= \lim_{x\to-\infty}\left(2x+\sqrt{4x^2+2x}\right)\cdot\frac{2x-\sqrt{4x^2+2x}}{2x-\sqrt{4x^2+2x}} = \lim_{x\to-\infty}\frac{4x^2-\left(4x^2+2x\right)}{2x-\sqrt{4x^2+2x}}$$

$$= \lim_{x\to-\infty}\frac{-2x}{2x-\sqrt{4x^2+2}} = \lim_{x\to-\infty}\frac{-2}{2-\frac{\sqrt{4x^2+2}}{x}}$$

$$= \frac{-2}{2-\lim_{x\to-\infty}\frac{\sqrt{4x^2+2}}{x}} = \frac{-2}{2-\lim_{x\to-\infty}\frac{\sqrt{4x^2+2}}{-\sqrt{\mathbf{x^2}}}}$$

$$= \frac{-2}{2+\lim_{x\to-\infty}\sqrt{4+\frac{2}{x^2}}} = \frac{-2}{2+\sqrt{4+\lim_{x\to-\infty}\frac{2}{x^2}}}$$

$$= \frac{-2}{2+\sqrt{4+0}} = \frac{-2}{2+2}$$

$$= \boxed{\frac{-1}{2}}$$

2. Find $\lim_{x\to 0}\frac{\tan(\sin(x))}{x}$
Hint: Try multiplying by something clever before substituting.

$$= \lim_{x\to 0}\frac{\sin(\sin(x))}{x}\cdot\frac{1}{\cos(\sin(x))} = \left(\lim_{x\to 0}\frac{\sin(\sin(x))}{x}\right)\cdot\left(\lim_{x\to 0}\frac{1}{\cos(\sin(x))}\right)$$

$$= \lim_{x\to 0}\frac{\sin(\sin(x))}{x}\cdot 1 = \lim_{x\to 0}\frac{\sin(\sin(x))}{x}\cdot\frac{\sin(x)}{\sin(x)}$$

$$= \lim_{x\to 0}\frac{\sin(\sin(x))}{\sin(x)}\cdot\frac{x}{\sin(x)} = \lim_{x\to 0}\frac{\sin(\sin(x))}{\sin(x)}\cdot\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right)^{-1}$$

If $z=\sin(x)$ then: $\lim_{x\to 0} z = 0$

$$= \left(\lim_{z\to 0}\frac{\sin(z)}{z}\right)\cdot 1$$

$$= \boxed{1}$$

3. Suppose $\lim_{x\to\infty} f(x)=\infty$, $\lim_{x\to\infty} g(x)=2$, $\lim_{x\to\infty} h(x)=0$

Calculate: $\lim_{x\to\infty} g(x)+\frac{1}{f(x)+h(x)}$

$\boxed{2}$

Calculus 1a
Quiz 3, Section: 109

Name:______________________________

Section 1. Short Answer (Show Your Work)

1. Find $\lim\limits_{x\to-\infty} 3x + \sqrt{9x^2 + 2x}$

2. Find $\lim\limits_{x\to 0} \frac{\tan(\sin(x))}{x}$
   Hint: Try multiplying by something clever before substituting.

3. Suppose $\lim\limits_{x\to 2} f(x) = \infty, \lim\limits_{x\to 2} g(x) = 2, \lim\limits_{x\to 2} h(x) = 0$

   Calculate: $\lim\limits_{x\to 2} \dfrac{f(x) + g(x)}{3 + f(x)} + \dfrac{h(x)}{f(x)}$

Answer Key for Section 109: Quiz 3

Section 1. Short Answer (Show Your Work)

1. Find $\lim\limits_{x\to-\infty} 3x+\sqrt{9x^2+2x}$

$$= \lim_{x\to-\infty}\left(3x+\sqrt{9x^2+2x}\right)\cdot\frac{3x-\sqrt{9x^2+2x}}{3x-\sqrt{9x^2+2x}} = \lim_{x\to-\infty}\frac{9x^2-\left(9x^2+2x\right)}{3x-\sqrt{9x^2+2x}}$$
$$= \lim_{x\to-\infty}\frac{-2x}{3x-\sqrt{9x^2+2}} = \lim_{x\to-\infty}\frac{-2}{3-\frac{\sqrt{9x^2+2}}{x}}$$
$$= \frac{-2}{3-\lim\limits_{x\to-\infty}\frac{\sqrt{9x^2+2}}{x}} = \frac{-2}{3-\lim\limits_{x\to-\infty}\frac{\sqrt{9x^2+2}}{-\sqrt{\mathbf{x^2}}}}$$
$$= \frac{-2}{3+\lim\limits_{x\to-\infty}\sqrt{9+\frac{2}{x^2}}} = \frac{-2}{3+\sqrt{9+\lim\limits_{x\to-\infty}\frac{2}{x^2}}}$$
$$= \frac{-2}{3+\sqrt{9+0}} = \frac{-2}{3+3}$$
$$= \boxed{\frac{-1}{3}}$$

2. Find $\lim\limits_{x\to0}\frac{\tan(\sin(x))}{x}$
Hint: Try multiplying by something clever before substituting.

$$= \lim_{x\to0}\frac{\sin(\sin(x))}{x}\cdot\frac{1}{\cos(\sin(x))} = \left(\lim_{x\to0}\frac{\sin(\sin(x))}{x}\right)\cdot\left(\lim_{x\to0}\frac{1}{\cos(\sin(x))}\right)$$
$$= \lim_{x\to0}\frac{\sin(\sin(x))}{x}\cdot1 = \lim_{x\to0}\frac{\sin(\sin(x))}{x}\cdot\frac{\sin(x)}{\sin(x)}$$
$$= \lim_{x\to0}\frac{\sin(\sin(x))}{\sin(x)}\cdot\frac{x}{\sin(x)} = \lim_{x\to0}\frac{\sin(\sin(x))}{\sin(x)}\cdot\lim_{x\to0}\left(\frac{\sin(x)}{x}\right)^{-1}$$

If $z=\sin(x)$ then: $\lim\limits_{x\to0} z = 0$

$$= \left(\lim_{z\to0}\frac{\sin(z)}{z}\right)\cdot1$$
$$= \boxed{1}$$

3. Suppose $\lim\limits_{x\to2} f(x)=\infty$, $\lim\limits_{x\to2} g(x)=2$, $\lim\limits_{x\to2} h(x)=0$

Calculate: $\lim\limits_{x\to2}\frac{f(x)+g(x)}{3+f(x)}+\frac{h(x)}{f(x)}$

$\boxed{1}$ (divide top and bottom of first fraction by $f(x)$)