

Section 1. Short Answer (Show Your Work)

1. Compute $\frac{d}{dx}[x^3 \cdot (f(g(x)))^2]$

2. Calculate $\frac{d}{dx}[\sin(x^4 \cos^2(x^2))]$

3. Find $f'(3)$ if $f(x) = g(x \cdot h(x))$ and $h(3) = 2, h(2) = 1, h'(2) = 2, h'(3) = 1, g'(6) = 2, g'(2) = 6$.

Answer Key for Section 109: Quiz 2

Section 1. Short Answer (Show Your Work)

1. Compute $\frac{d}{dx}[x^3 \cdot (f(g(x)))^2]$

$$\begin{aligned} &= \frac{d}{dx}[x^3] \cdot (f(g(x)))^2 + x^3 \cdot \frac{d}{dx}[(f(g(x)))^2] \\ &= 3x^2 \cdot (f(g(x)))^2 + x^3 \cdot 2(f(g(x))) \cdot \frac{d}{dx}[f(g(x))] \\ &= 3x^2 \cdot (f(g(x)))^2 + 2x^3 \cdot (f(g(x))) \cdot f'(g(x)) \cdot g'(x) \end{aligned}$$

2. Calculate $\frac{d}{dx}[\sin(x^4 \cos^2(x^2))]$

$$\begin{aligned} &= \cos(x^4 \cos^2(x^2)) \cdot \frac{d}{dx}[x^4 \cos^2(x^2)] \\ &= \cos(x^4 \cos^2(x^2)) \cdot \left(\frac{d}{dx}[x^4] \cos^2(x^2) + x^4 \frac{d}{dx}[\cos^2(x^2)] \right) \\ &= \cos(x^4 \cos^2(x^2)) \cdot \left(4x^3 \cos^2(x^2) + 2x^4 \cos(x^2) \frac{d}{dx}[\cos(x^2)] \right) \\ &= \cos(x^4 \cos^2(x^2)) \cdot \left(4x^3 \cos^2(x^2) + 2x^4 \cos(x^2) (-\sin(x^2)) \frac{d}{dx}[x^2] \right) \\ &= \cos(x^4 \cos^2(x^2)) \cdot (4x^3 \cos^2(x^2) - 4x^4 \cos(x^2) \sin(x^2)x) \\ &= \cos(x^4 \cos^2(x^2)) \cdot (4x^3 \cos^2(x^2) - 4x^5 \cos(x^2) \sin(x^2)) \end{aligned}$$

3. Find $f'(3)$ if $f(x) = g(x \cdot h(x))$ and $h(3) = 2, h(2) = 1, h'(2) = 2, h'(3) = 1, g'(6) = 2, g'(2) = 6$.

$$\begin{aligned} f'(x) &= g'(x \cdot h(x)) \cdot [h(x) + x \cdot h'(x)] \\ f'(3) &= g'(3 \cdot h(3)) \cdot [h(3) + 3 \cdot h'(3)] \\ f'(3) &= g'(3 \cdot 2) \cdot [2 + 3 \cdot 1] \\ f'(3) &= g'(6) \cdot 5 \\ f'(3) &= 10 \end{aligned}$$

Section 1. Short Answer (Show Your Work)

1. Compute $\frac{d}{dx}[x^2 \cdot (f(g(x)))^3]$

2. Calculate $\frac{d}{dx}[\sin(x^3 \cos^2(x^4))]$

3. Find $f'(2)$ if $f(x) = g(x \cdot h(x))$ and $h(3) = 2, h(2) = 1, h'(2) = 2, h'(3) = 1, g'(6) = 2, g'(2) = 6$.

Answer Key for Section 106: Quiz 2

Section 1. Short Answer (Show Your Work)

1. Compute $\frac{d}{dx}[x^2 \cdot (f(g(x)))^3]$

$$\begin{aligned} &= \frac{d}{dx}[x^2] \cdot (f(g(x)))^3 + x^2 \cdot \frac{d}{dx}[(f(g(x)))^3] \\ &= 2x \cdot (f(g(x)))^3 + x^2 \cdot 3(f(g(x)))^2 \cdot \frac{d}{dx}[f(g(x))] \\ &= 2x \cdot (f(g(x)))^3 + 3x^2 \cdot (f(g(x)))^2 \cdot f'(g(x)) \cdot g'(x) \end{aligned}$$

2. Calculate $\frac{d}{dx}[\sin(x^3 \cos^2(x^4))]$

$$\begin{aligned} &= \cos(x^3 \cos^2(x^4)) \cdot \frac{d}{dx}[x^3 \cos^2(x^4)] \\ &= \cos(x^3 \cos^2(x^4)) \cdot \left(\frac{d}{dx}[x^3] \cos^2(x^4) + x^3 \frac{d}{dx}[\cos^2(x^4)] \right) \\ &= \cos(x^3 \cos^2(x^4)) \cdot \left(3x^2 \cos^2(x^4) + 2x^3 \cos(x^4) \frac{d}{dx}[\cos(x^4)] \right) \\ &= \cos(x^3 \cos^2(x^4)) \cdot \left(3x^2 \cos^2(x^4) + 2x^3 \cos(x^4) (-\sin(x^4)) \frac{d}{dx}[x^4] \right) \\ &= \cos(x^3 \cos^2(x^4)) \cdot (3x^2 \cos^2(x^4) - 8x^3 \cos(x^4) \sin(x^4)x^3) \\ &= \cos(x^3 \cos^2(x^4)) \cdot (3x^2 \cos^2(x^4) - 8x^6 \cos(x^4) \sin(x^4)) \end{aligned}$$

3. Find $f'(2)$ if $f(x) = g(x \cdot h(x))$ and $h(3) = 2, h(2) = 1, h'(2) = 2, h'(3) = 1, g'(6) = 2, g'(2) = 6$.

$$\begin{aligned} f'(x) &= g'(x \cdot h(x)) \cdot [h(x) + x \cdot h'(x)] \\ f'(2) &= g'(2 \cdot h(2)) \cdot [h(2) + 2 \cdot h'(2)] \\ f'(2) &= g'(2 \cdot 1) \cdot [1 + 2 \cdot 2] \\ f'(2) &= g'(2) \cdot 5 \\ f'(2) &= 30 \end{aligned}$$